

Deep machines that know when they do not know

and how to exploit symmetries for modelling and solving quadratic programs



Kristian Kersting



Neural Networks
a mostly complete cheat sheet of
©2016 Fjodor van Veen - adam@informatik.uni-dm.de

- Backfed Input Cell
- Input Cell
- Noisy Input Cell
- Hidden Cell
- Probabilistic Cell
- Spike Cell
- Output Cell
- Max Cell
- Min Cell

Perceptron (P) Feed Forward (FF) Radial Basis Network (RBF) Deep Feed Forward (DFF)

Long / Short Term Memory (LSTM) Gated Recurrent Unit (GRU)

... convex sum
... product
... distribution

completeness
uniform children: same scope

decomposability
product children: non-overlapping scope

$\{X_1, X_2\}$
 $\{A_1, A_2, A_3\}$
 $\{X_3\}$

```
#inline definitions  
#slacks = sum{I in labeled(I)} slack(I);  
  
#QUADRATIC OBJECTIVE  
#minimize: sum{J in feature(I,J)} weight(J)**2 + c1 * slack;  
#labeled examples should be on the correct side  
#subject to forall {I in labeled(I)}: labeled(I)*predict(I) >= 1 - slack  
#slacks are non-negative  
#subject to forall {I in labeled(I)}: slack(I) >= 0;
```

(a) (b) (c)

AI and ML have a strong impact

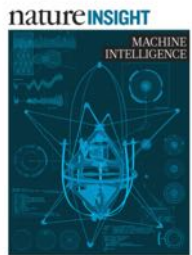


Data are now ubiquitous; there is great value from understanding this data, building models and making predictions

However, there are not enough data scientists, statisticians, machine learning and AI experts

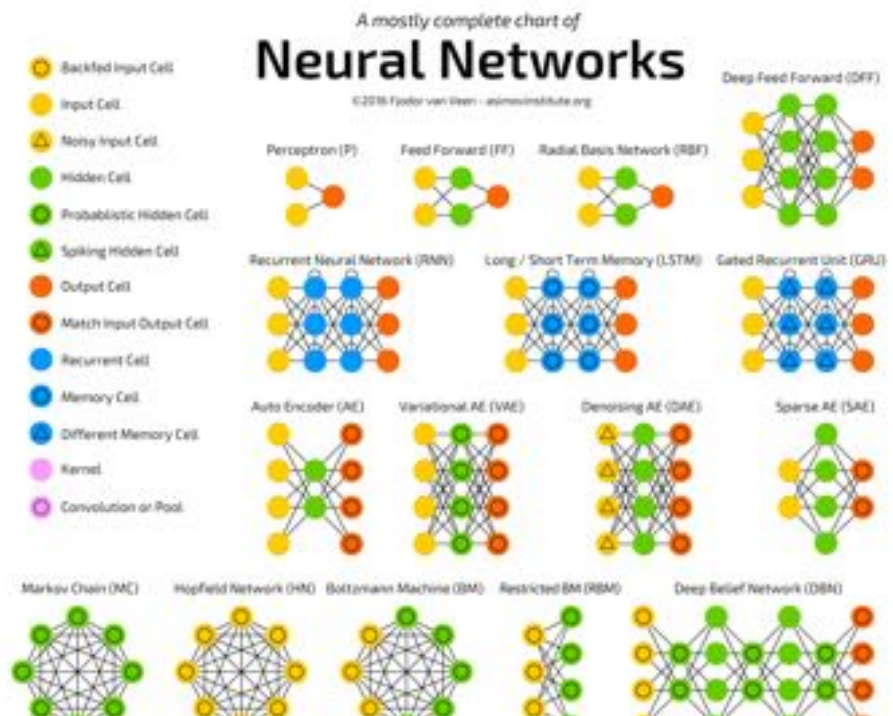
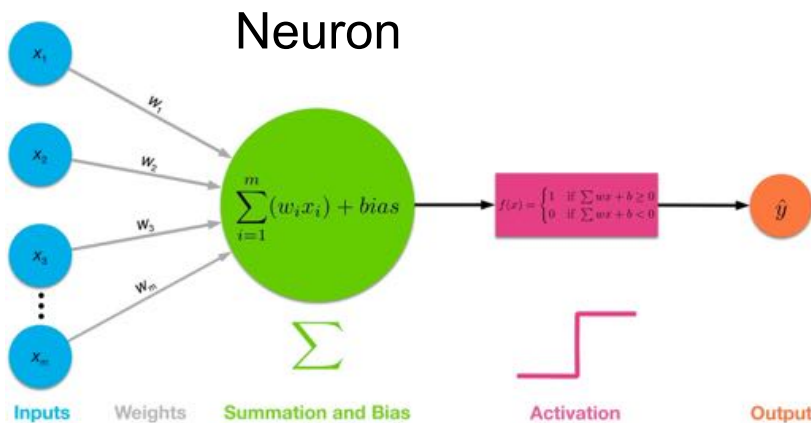
Provide the foundations, algorithms, and tools to develop systems that ease or even automate AI model discovery from data as much as possible

Deep Neural Networks



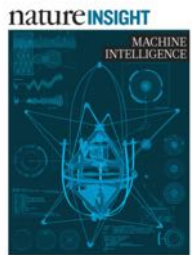
Potentially much more powerful than shallow architectures, represent computations

[LeCun, Bengio, Hinton Nature 521, 436–444, 2015]



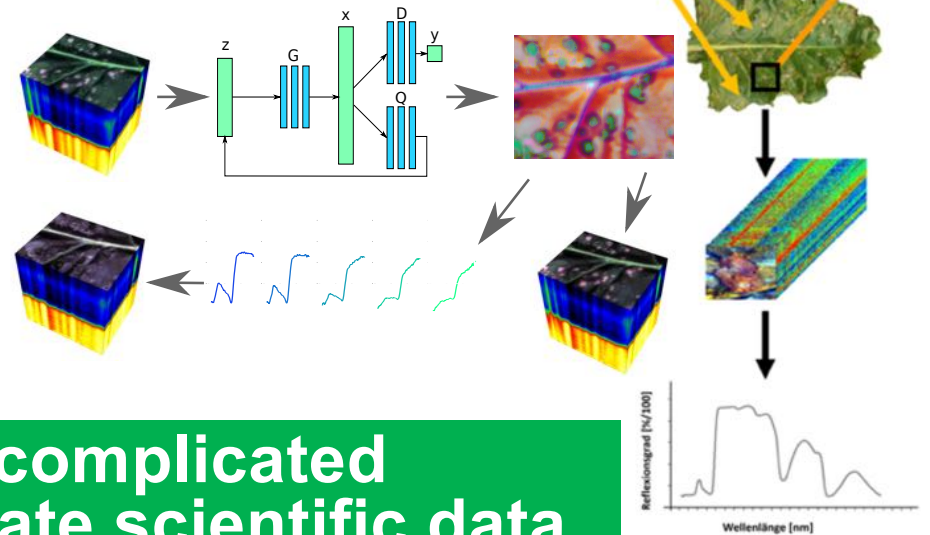
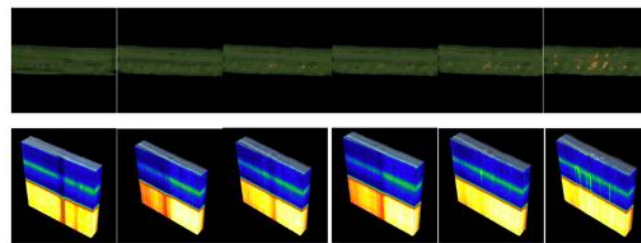
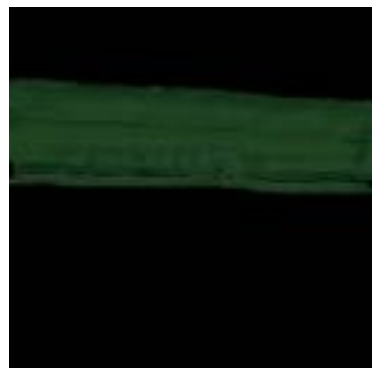
Differentiable Programming

Deep Neural Networks



Potentially much more powerful than shallow architectures, represent computations

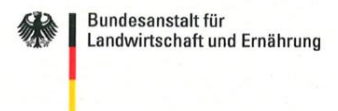
[LeCun, Bengio, Hinton Nature 521, 436–444, 2015]



They “develop intuition” about complicated biological processes and generate scientific data

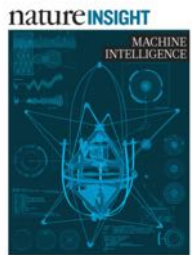
[Schramowski, Brugger, Mahlein, Kersting 2019]

DePhenSe



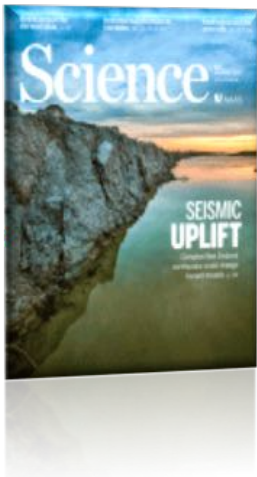


Deep Neural Networks



Potentially much more powerful than shallow architectures, represent computations

[LeCun, Bengio, Hinton Nature 521, 436–444, 2015]



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1.02k



0

Semantics derived automatically from language corpora contain human-like biases

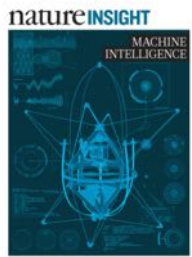
Aylin Caliskan^{1,*}, Joanna J. Bryson^{1,2,*}, Arvind Narayanan^{1,*}

+ See all authors and affiliations

Science 14 Apr 2017:
Vol. 356, Issue 6334, pp. 183-186
DOI: 10.1126/science.124230

They “capture” stereotypes from human language

Deep Neural Networks



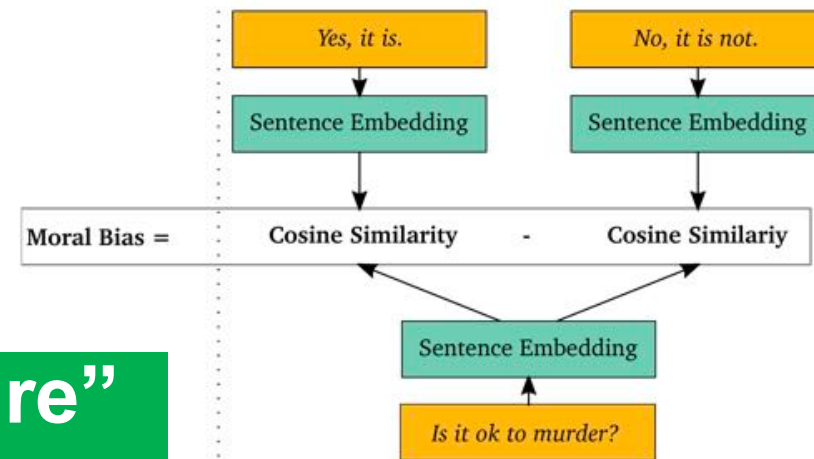
Potentially much more powerful than shallow architectures, represent computations

[LeCun, Bengio, Hinton Nature 521, 436–444, 2015]

The Moral Choice Machine

Dos	WEAT	Bias	Don'ts	WEAT	Bias
smile	0.116	0.348	rot	-0.099	-1.118
sightsee	0.090	0.281	negative	-0.101	-0.763
cheer	0.094	0.277	harm	-0.110	-0.730
celebrate	0.114	0.264	damage	-0.105	-0.664
picnic	0.093	0.260	slander	-0.108	-0.600
snuggle	0.108	0.238	slur	-0.109	-0.569

But lucky they also “capture” our moral choices



[Jentzsch, Schramowski, Rothkopf, Kersting AIES 2019]



AAAI / ACM conference on
**ARTIFICIAL INTELLIGENCE,
ETHICS, AND SOCIETY**

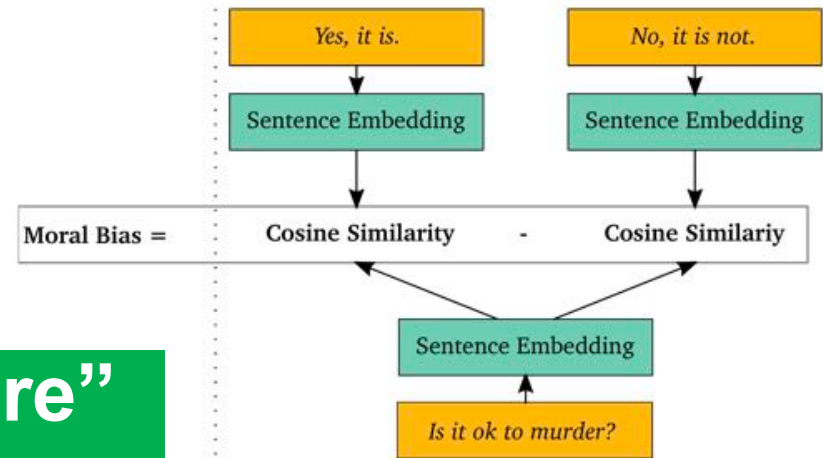


Video 05:10 Min.
 Der Hamster gehört nicht in den Toaster – Wie Forscher von der TU Darmstadt versuchen, Maschinen ... [Videoseite]
 hauptsache kultur | 14.03.19, 22:45 Uhr

The Moral Choice Machine

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 ETHICS, AND SOCIETY**

Deep neural networks do not quantify their uncertainty They are not calibrated probabilistic models

MNIST



Train & Evaluate

SVHN

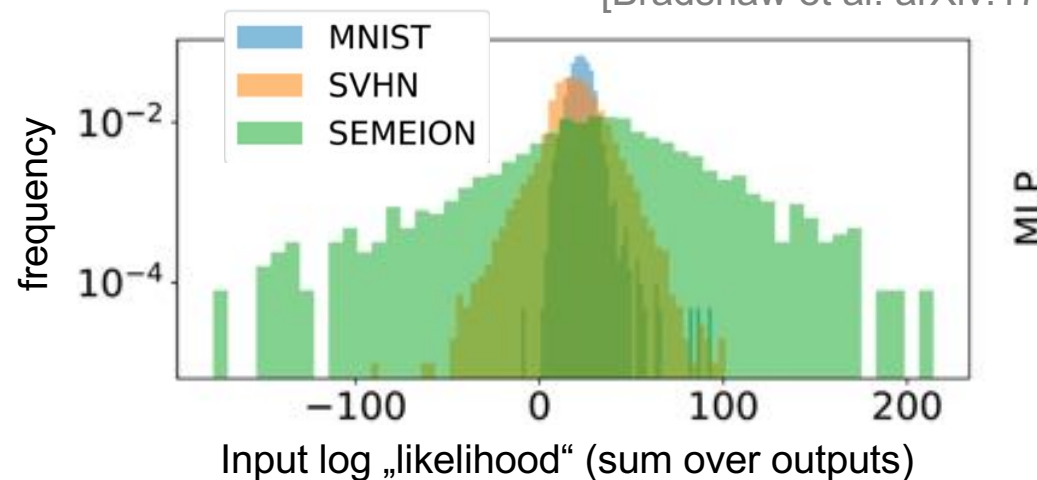


SEMEION



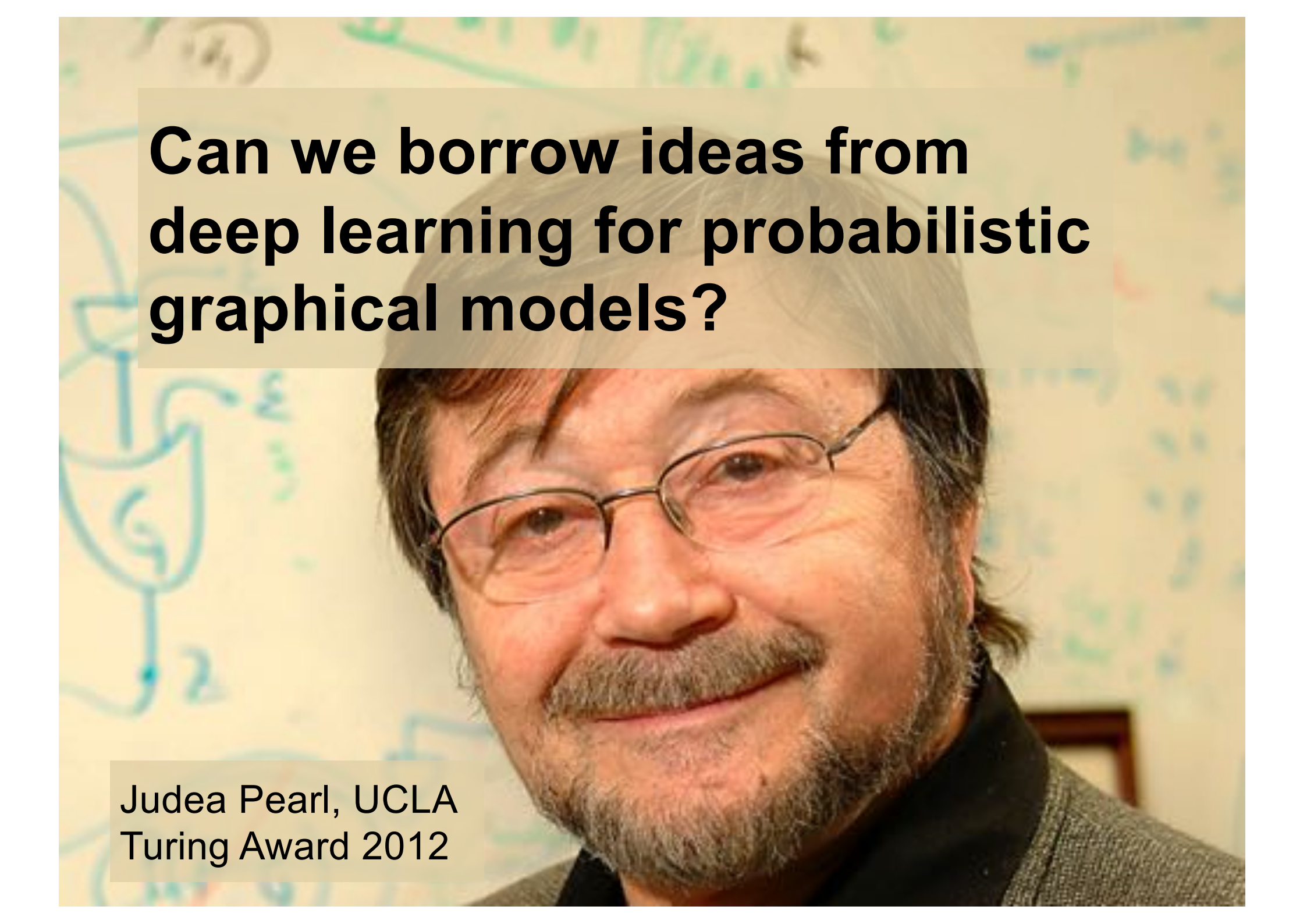
Transfer Testing

[Bradshaw et al. arXiv:1707.02476 2017]



[Peharz, Vergari, Molina, Stelzner, Trapp, Kersting, Ghahramani UDL@UAI 2018]

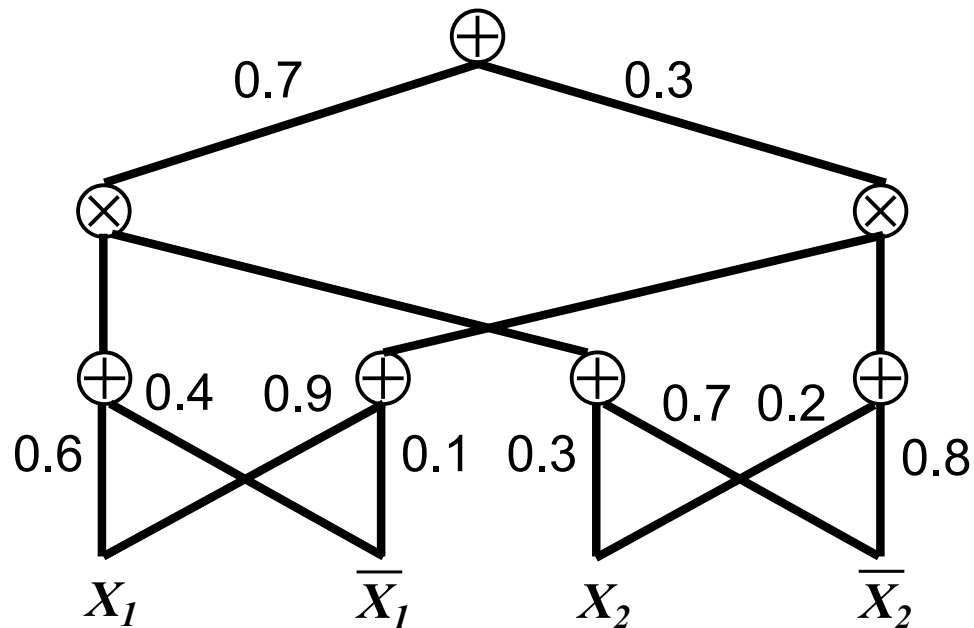
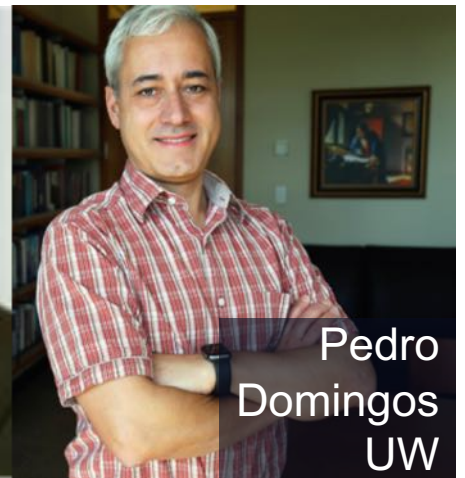
**Getting deep systems that know
when they don't know.**



Can we borrow ideas from deep learning for probabilistic graphical models?

Judea Pearl, UCLA
Turing Award 2012

This results in Sum-Product Networks, a deep probabilistic learning framework



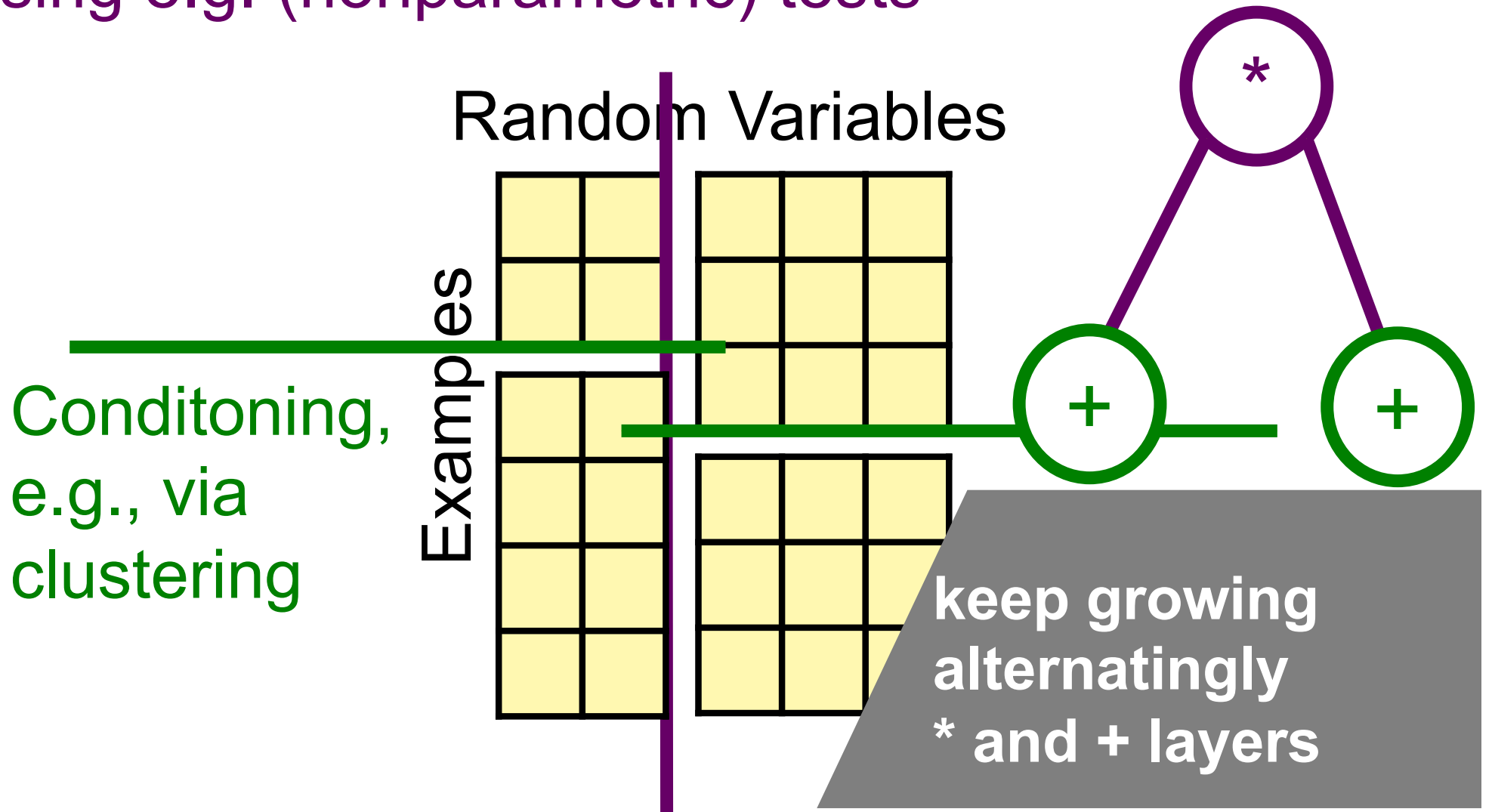
Computational graph
(kind of TensorFlow
graphs) that encodes
how to compute
probabilities

Inference is linear in size of network



And there is a way to select models

Testing independence of random variables using e.g. (nonparametric) tests



[Poon, Domingos UAI'11; Molina, Natarajan, Kersting AAAI'17; Vergari, Peharz, Di Mauro, Molina, Kersting, Esposito AAAI '18; Molina, Vergari, Di Mauro, Esposito, Natarajan, Kersting AAAI '18]

FL ⊕ W for SPFlow: An Easy and Extensible Library for Sum-Product Networks

[Molina, Vergari, Stelzner, Peharz, Subramani, Poupart, Di Mauro, Kersting 2019]



<https://github.com/SPFlow/SPFlow>

```
from spn.structure.leaves.parametric.Parametric import Categorical
from spn.structure.Base import Sum, Product
from spn.structure.base import assign_ids, rebuild_scopes_bottom_up

p0 = Product(children=[Categorical(p=[0.3, 0.7], scope=1), Categorical(p=[0.4, 0.6], scope=2)])
p1 = Product(children=[Categorical(p=[0.5, 0.5], scope=1), Categorical(p=[0.6, 0.4], scope=2)])
s1 = Sum(weights=[0.3, 0.7], children=[p0, p1])
p2 = Product(children=[Categorical(p=[0.2, 0.8], scope=0), s1])
p3 = Product(children=[Categorical(p=[0.2, 0.8], scope=0), Categorical(p=[0.3, 0.7], scope=1)])
p4 = Product(children=[p3, Categorical(p=[0.4, 0.6], scope=2)])
spn = Sum(weights=[0.4, 0.6], children=[p2, p4])

assign_ids(spn)
rebuild_scopes_bottom_up(spn)

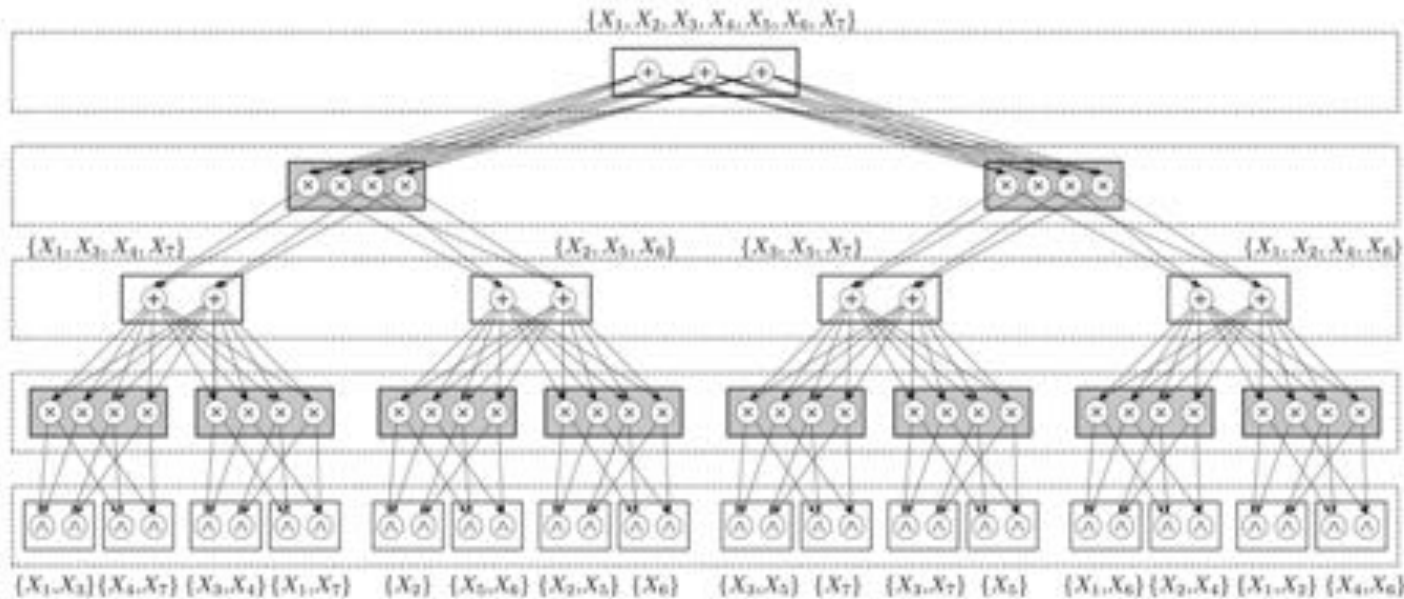
return spn
```

Domain Specific Language, Inference, EM, and Model Selection as well as Compilation of SPNs into TF and PyTorch and also into flat, library-free code even suitable for running on devices: C/C++, GPU, FPGA

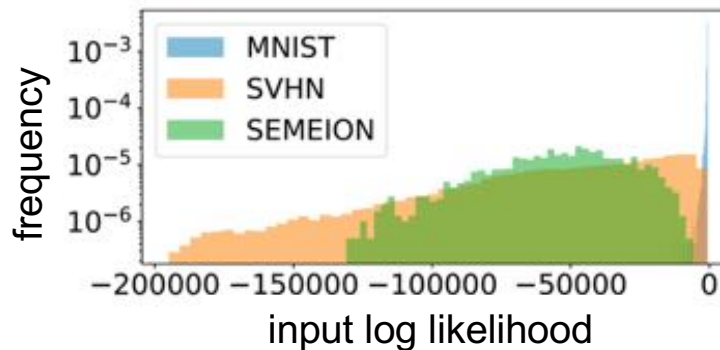
SPFlow, an open-source Python library providing a simple interface to inference, learning and manipulation routines for deep and tractable probabilistic models called Sum-Product Networks (SPNs). The library allows one to quickly create SPNs both from data and through a domain specific language (DSL). It efficiently implements several probabilistic inference routines like computing marginals, conditionals and (approximate) most probable explanations (MPEs) along with compilation

Random sum-product networks

[Peharz, Vergari, Molina, Stelzner, Trapp, Kersting, Ghahramani UDL@UAI 2018]



	RAT-SPN	MLP	vMLP	
Accuracy	MNIST	98.19 (8.5M)	98.32 (2.64M)	98.09 (5.28M)
	F-MNIST	89.52 (0.65M)	90.81 (9.28M)	89.81 (1.07M)
	20-NG	47.8 (0.37M)	49.05 (0.31M)	48.81 (0.16M)
Cross-Entropy	MNIST	0.0852 (17M)	0.0874 (0.82M)	0.0974 (0.22M)
	F-MNIST	0.3525 (0.65M)	0.2965 (0.82M)	0.325 (0.29M)
	20-NG	1.6954 (1.63M)	1.6180 (0.22M)	1.6263 (0.22M)



Learning the Structure of Autoregressive Deep Models such as PixelCNNs [van den Oord et al. NIPS 2016]



Learn Conditional SPN by testing conditional independence and using conditional clustering, using e.g.
[Zhang et al. UAI 2011; Lee, Honovar UAI 2017; He et al. ICDM 2017; Zhang et al. AAI 2018; Runge AISTATS 2018]

Conditional SPNs

[Shao, Molina, Vergari, Pecharz, Kersting 2019]

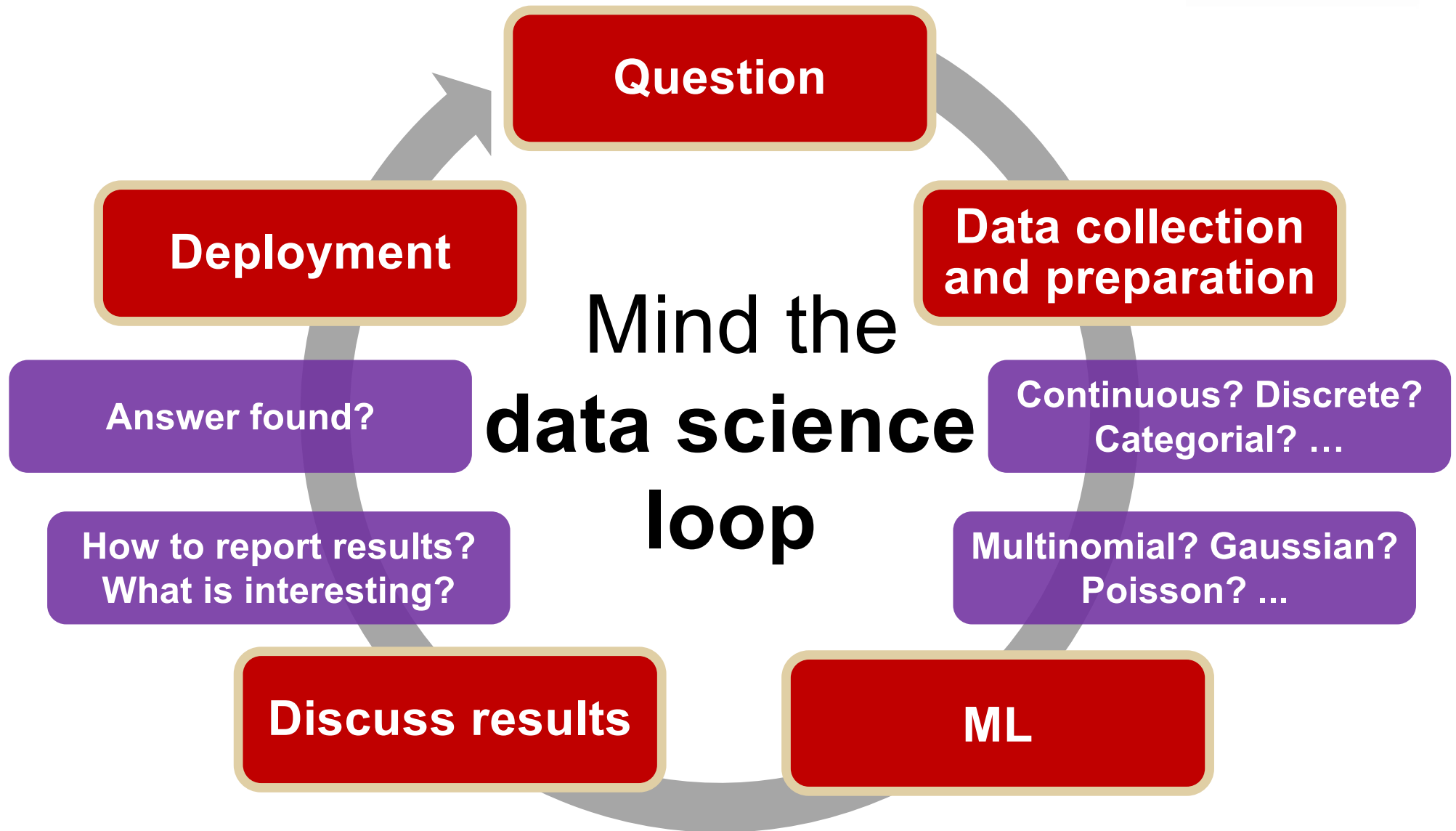
Functional weights realized as neural network



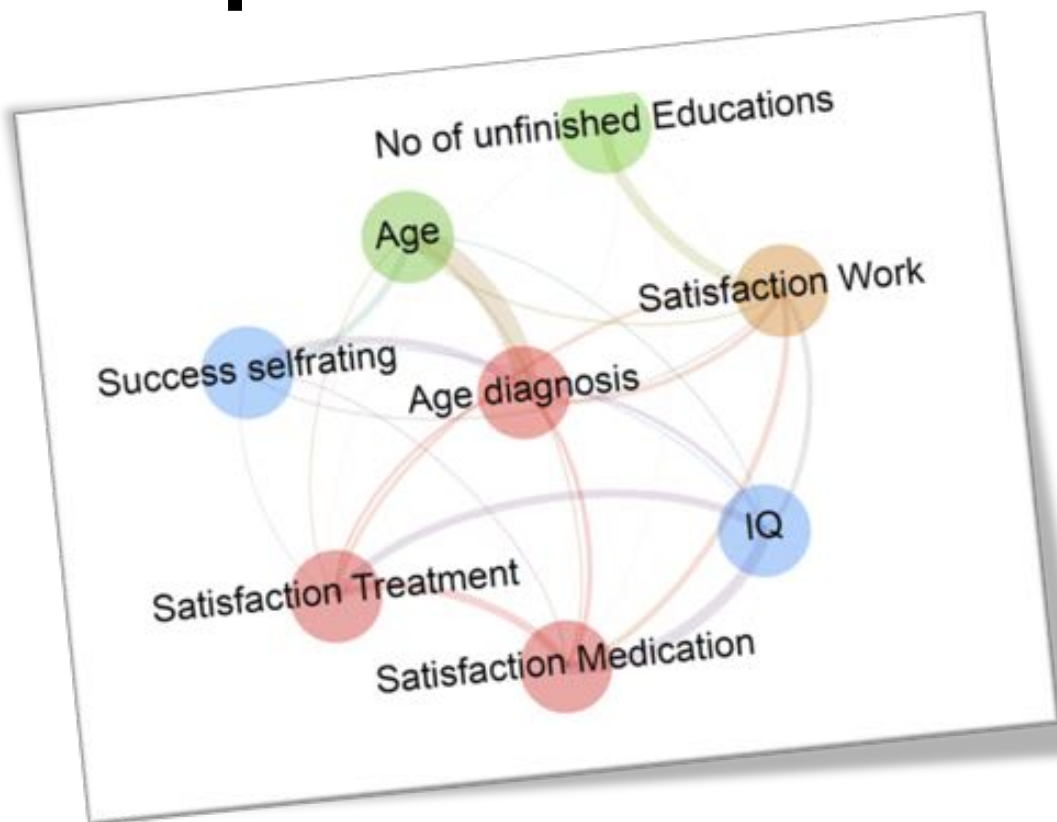
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Conditional SPNs

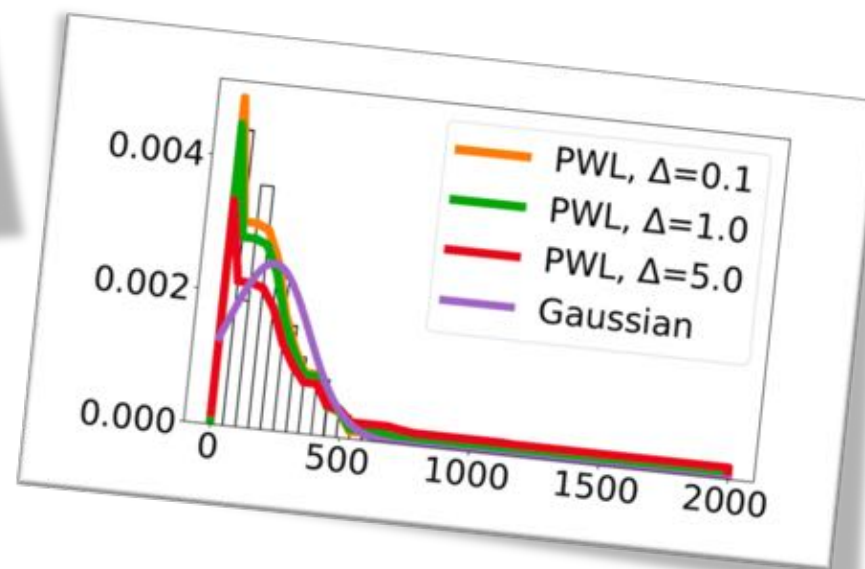
[Shao, Molina, Vergari, Pecharz, Kersting 2019]



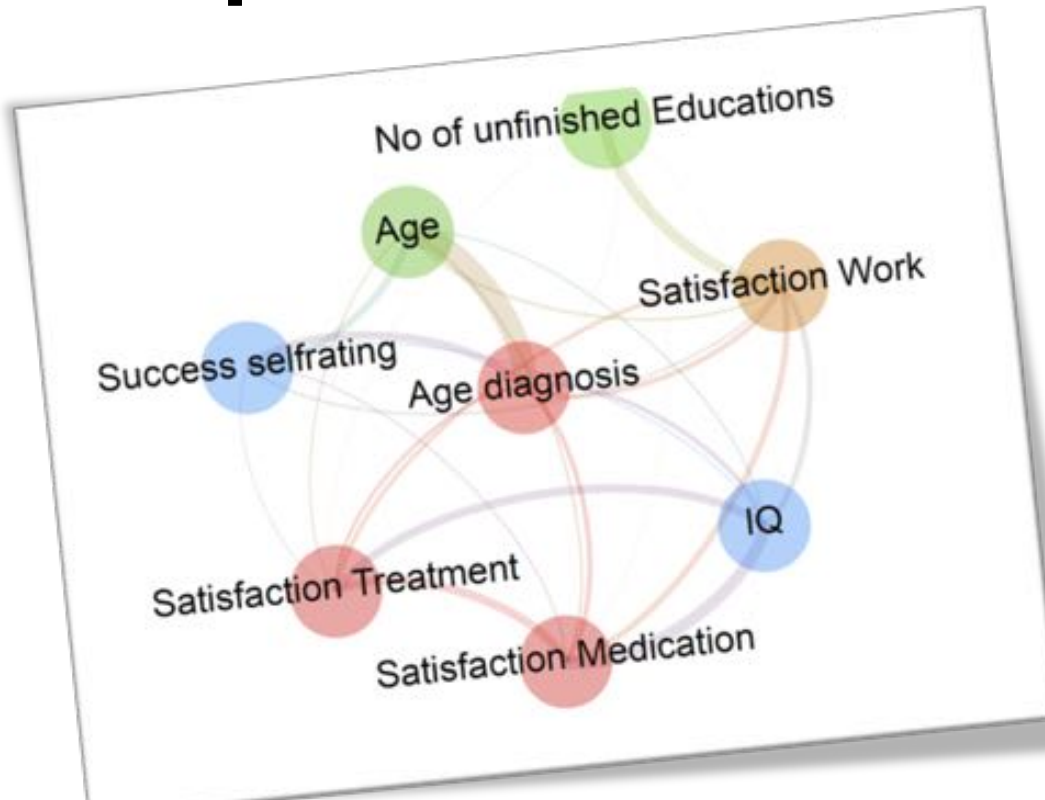
Distribution-agnostic Deep Probabilistic Learning



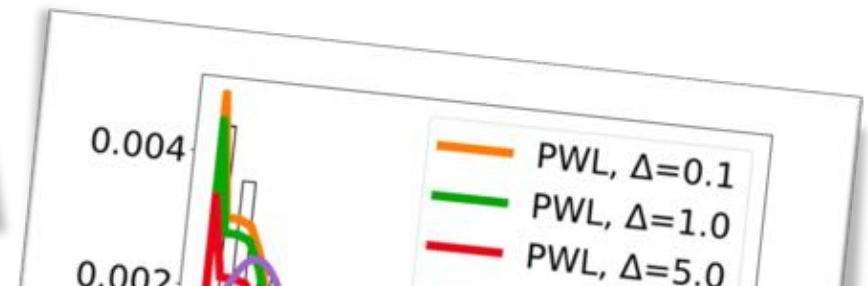
**Use nonparametric
independency tests
and piece-wise linear
approximations**



Distribution-agnostic Deep Probabilistic Learning



**Use nonparametric
independency tests
and piece-wise linear
approximations**



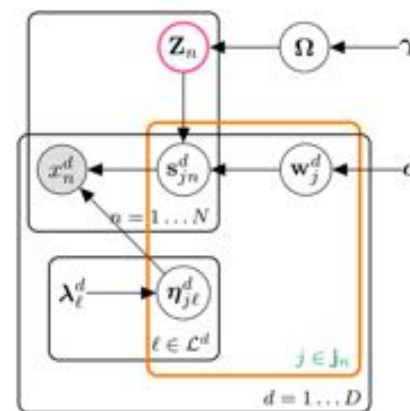
However, we have to provide the statistical types and do not gain insights into the parametric forms of the variables.
Are they Gaussians? Gammas? ...

The Explorative Automatic Statistician

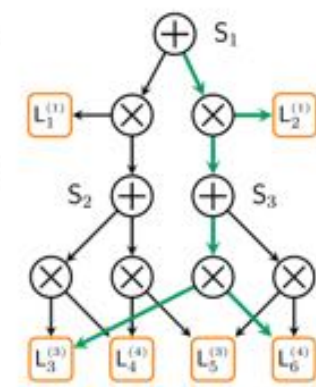


	X^1	X^2	X^3	X^4	X^5
x_6					
x_7			?		
x_8					
missing value x_9	?				
x_4				?	
x_3					
x_2		?			
x_1					

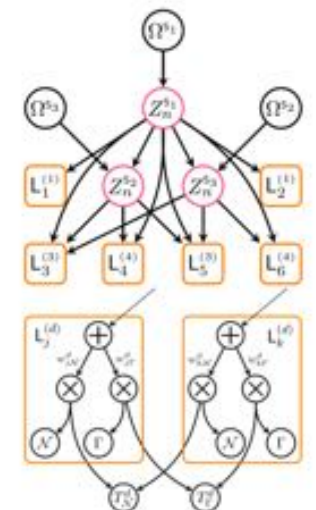
We can even automatically discovers the statistical types and parametric forms of the variables



Bayesian Type Discovery



Mixed Sum-Product Network



Automatic Statistician

That is, the machine understands the data with few expert input ...

The screenshot shows a Jupyter Notebook interface with three toggle buttons at the top: "Toggle Introduction", "Toggle explanations", and "Toggle Code". The main content is a report titled "Exploring the Titanic dataset". The report text describes the Titanic dataset and contains general statistical information and an analysis on the influence of different features and subgroups. It mentions that the first part contains general statistical information and an analysis of variables and probability distributions, while the second part focuses on a subgroup analysis. The report is generated by fitting a sum product network to the data and extracting all information from this model.

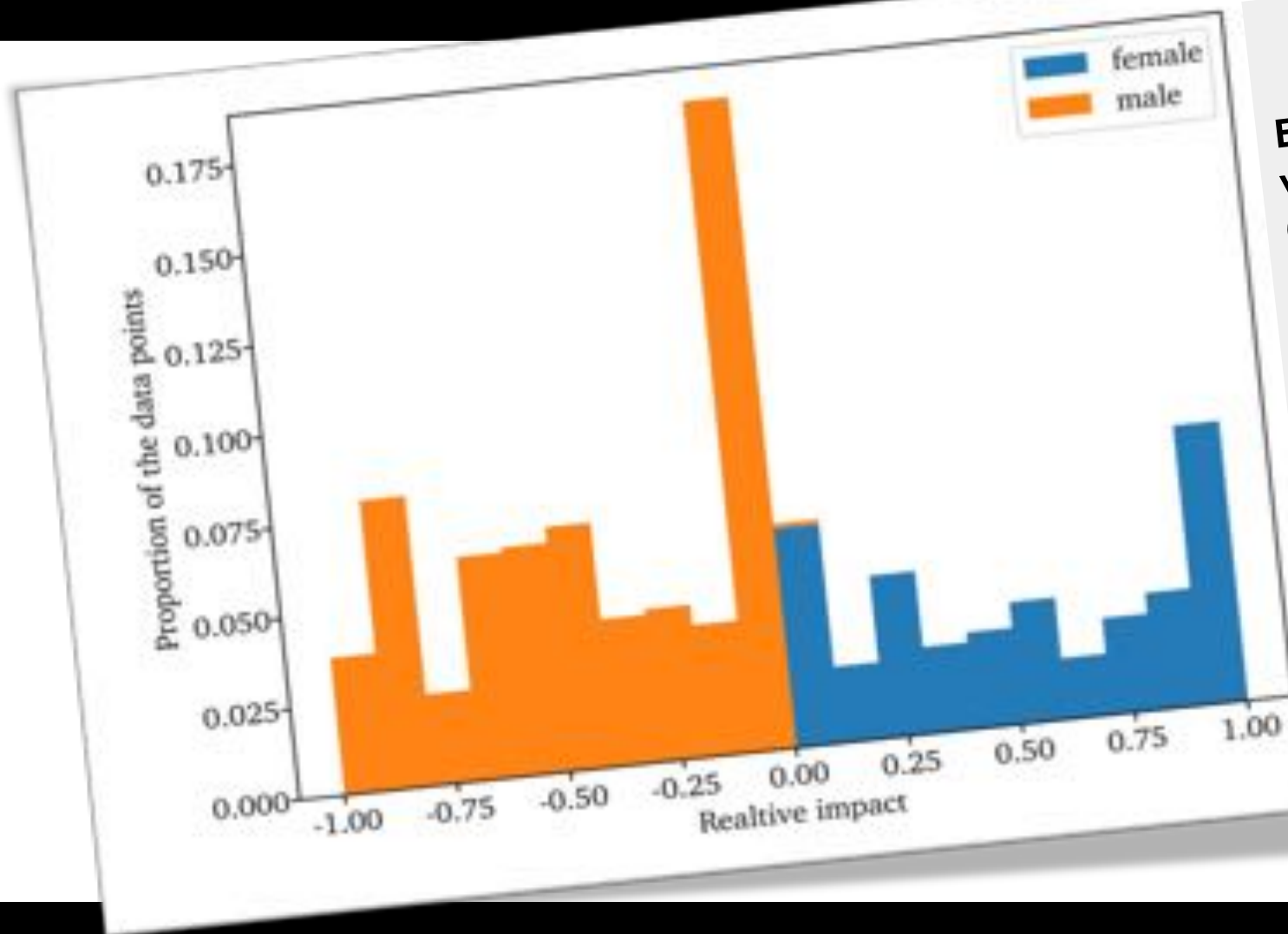
Völker: "DeepNotebooks – Interactive data analysis using Sum-Product Networks." MSc Thesis, TU Darmstadt, 2018

TECHNISCHE UNIVERSITÄT DARMSTADT
Report framework created @ TU Darmstadt

...and can compile data reports automatically

*[Baehrens, Schroeter, Harmeling, Kawanabe, Hansen, Müller JMLR 11:1803-1831, 2010]

The machine understands the data with no expert input ...



Explanation vector*
(computable in linear time in the size of the SPN) showing the impact of "gender" on the chances of survival for the Titanic dataset

...and can compile data reports automatically

P(heart attack | )?

The New York Times

Opinion

A.I. Is Harder Than You Think
and Data Science

By Gary Marcus and Ernest Davis

Mr. Marcus is a professor of psychology and neural science. Mr. Davis is a professor of computer science.

May 18, 2018



P(heart attack | )?

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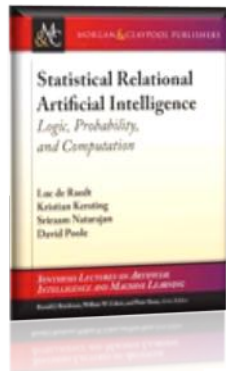
May 18, 2018

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P(heart attack |)?

Crossover of ML and DS with data & programming abstractions

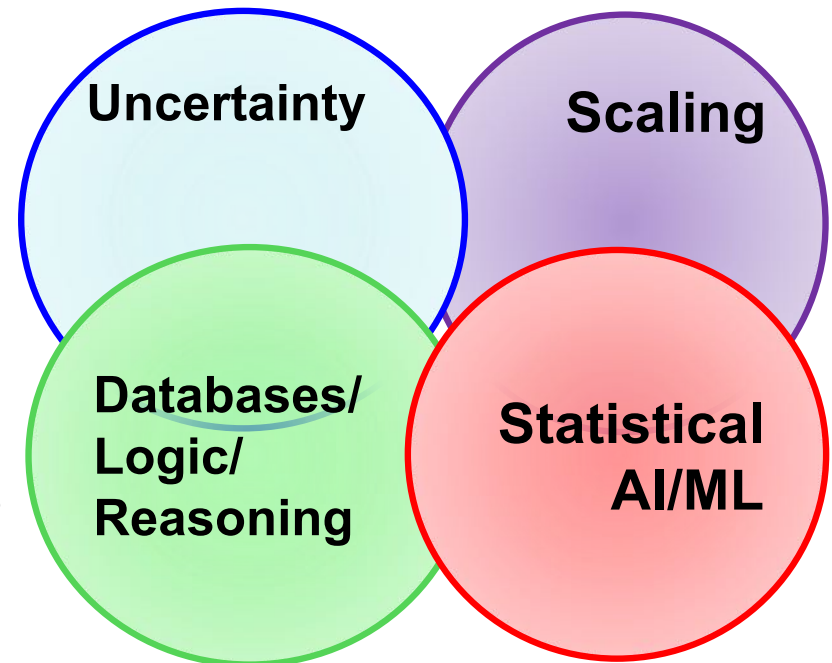
De Raedt, Kersting, Natarajan, Poole: Statistical Relational Artificial Intelligence: Logic, Probability, and Computation. Morgan and Claypool Publishers, ISBN: 9781627058414, 2016.

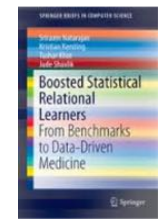


building general-purpose data science and ML machines

make the ML/DS expert more effective

increases the number of people who can successfully build ML/DS applications





Understanding Electronic Health Records

Atherosclerosis is the cause of the majority of Acute Myocardial Infarctions (heart attacks)



TECHNISCHE UNIVERSITÄT DARMSTADT

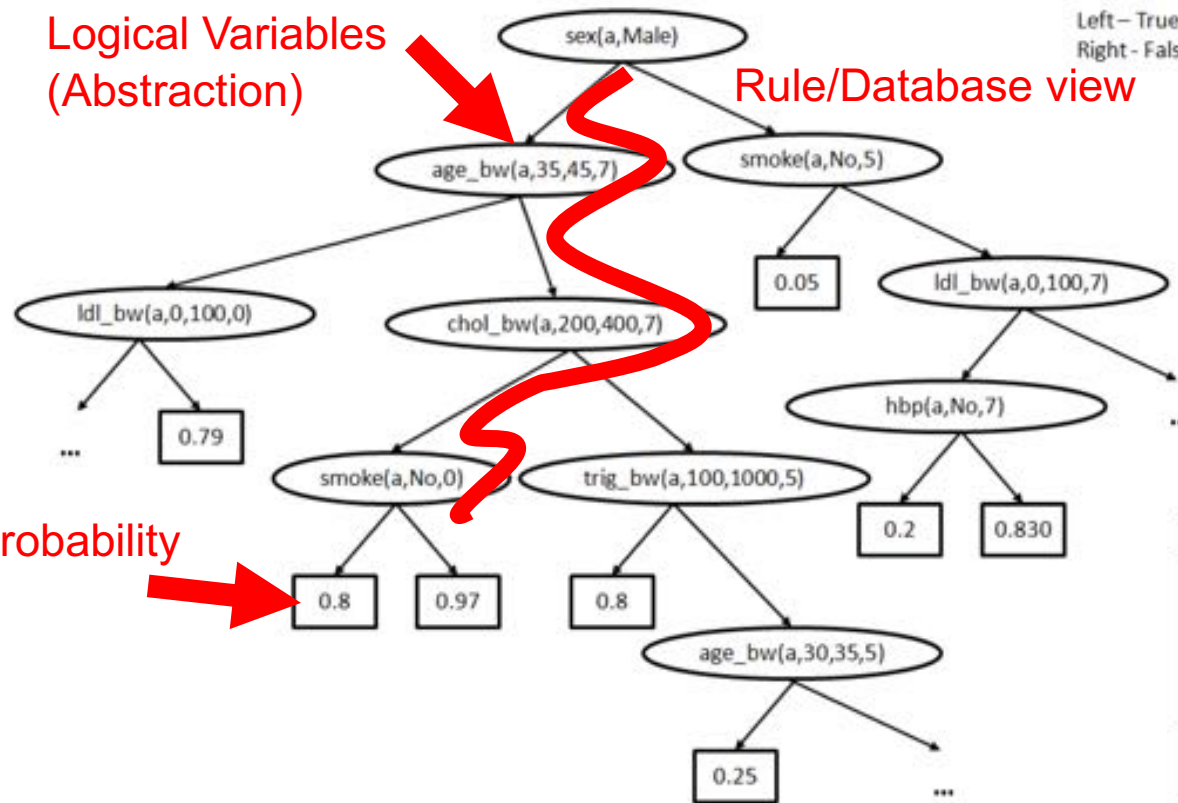


THE UNIVERSITY OF TEXAS AT DALLAS

Logical Variables (Abstraction)

Rule/Database view

Left - True
Right - False



Plaque in the left coronary artery

[Circulation; 92(8), 2157-62, 1995; JACC; 43, 842-7, 2004]

Probability

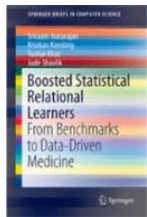
Algorithm	Accuracy	AUC-ROC
J48	0.667	0.607
SVM	0.667	0.5
AdaBoost	0.667	0.608
Bagging	0.677	0.613
NB	0.75	0.653
RPT	0.669*	0.778
RFGB	0.667*	0.819

The higher, the better

25%

Algorithm for Mining Markov Logic Networks	Likelihood The higher, the better	AUC-ROC The higher, the better	AUC-PR The higher, the better	Time The lower, the better	state-of-the-art
Boosting	0.81] 11%	0.96] 78%	0.93] 50%	9s] 37200x	
LSM	0.73]	0.54]	0.62]	93 hrs] faster	

[Kersting, Driessens ICML'08; Karwath, Kersting, Landwehr ICDM'08; Natarajan, Joshi, TadePELLI, Kersting, Shavlik. IJCAI'11; Natarajan, Kersting, Ip, Jacobs, Carr IAAI'13; Yang, Kersting, Terry, Carr, Natarajan AIME'15; Khot, Natarajan, Kersting, Shavlik ICDM'13, MLJ'12, MLJ'15, Yang, Kersting, Natarajan BIBM'17]



<https://starling.utdallas.edu/software/boostsrl/wiki/>



People

Publications

Projects

Software

Datasets

Blog



BOOSTSRL BASICS

- Getting Started
- File Structure
- Basic Parameters
- Advanced Parameters
- Basic Modes
- Advanced Modes

ADVANCED BOOSTSRL

- Default (RDN-Boost)
- MLN-Boost
- Regression
- One-Class Classification
- Cost-Sensitive SRL
- Learning with Advice
- Approximate Counting
- Discretization of Continuous-Valued Attributes
- Lifted Relational Random Walks
- Grounded Relational Random Walks

APPLICATIONS

- Natural Language Processing

BoostSRL Wiki

BoostSRL (Boosting for Statistical Relational Learning) is a gradient-boosting based approach to learning different types of SRL models. As with the standard gradient-boosting approach, our approach turns the model learning problem to learning a sequence of regression models. The key difference to the standard approaches is that we learn relational regression models i.e., regression models that operate on relational data. We assume the data in a predicate logic format and the output are essentially first-order regression trees where the inner nodes contain conjunctions of logical predicates. For more details on the models and the algorithm, we refer to our book on this topic.

Sriraam Natarajan, Tushar Khot, Kristian Kersting and Jude Shavlik, Boosted Statistical Relational Learners: From Benchmarks to Data-Driven Medicine . SpringerBriefs in Computer Science, ISBN: 978-3-319-13643-1, 2015

Human-in-the-loop learning

A simple example



Guy van den Broeck
UCLA

What is the problem that the first card of a randomly shuffled deck with 52 cards is an Ace?

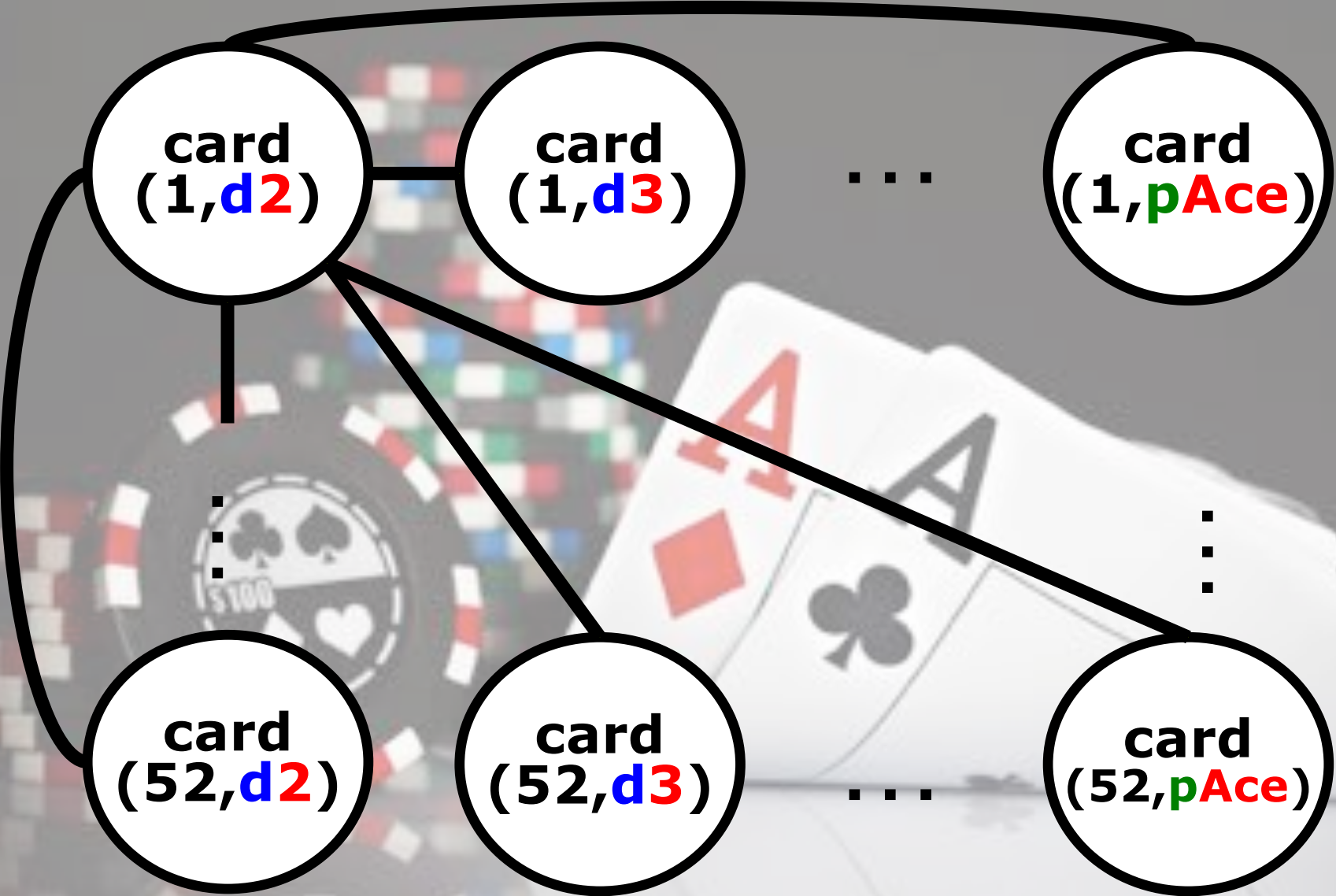
How would a machine solve this?

One option is to treat this as an inference problem within in a graphical model, solved approximately using some mathematical program!

A simple example



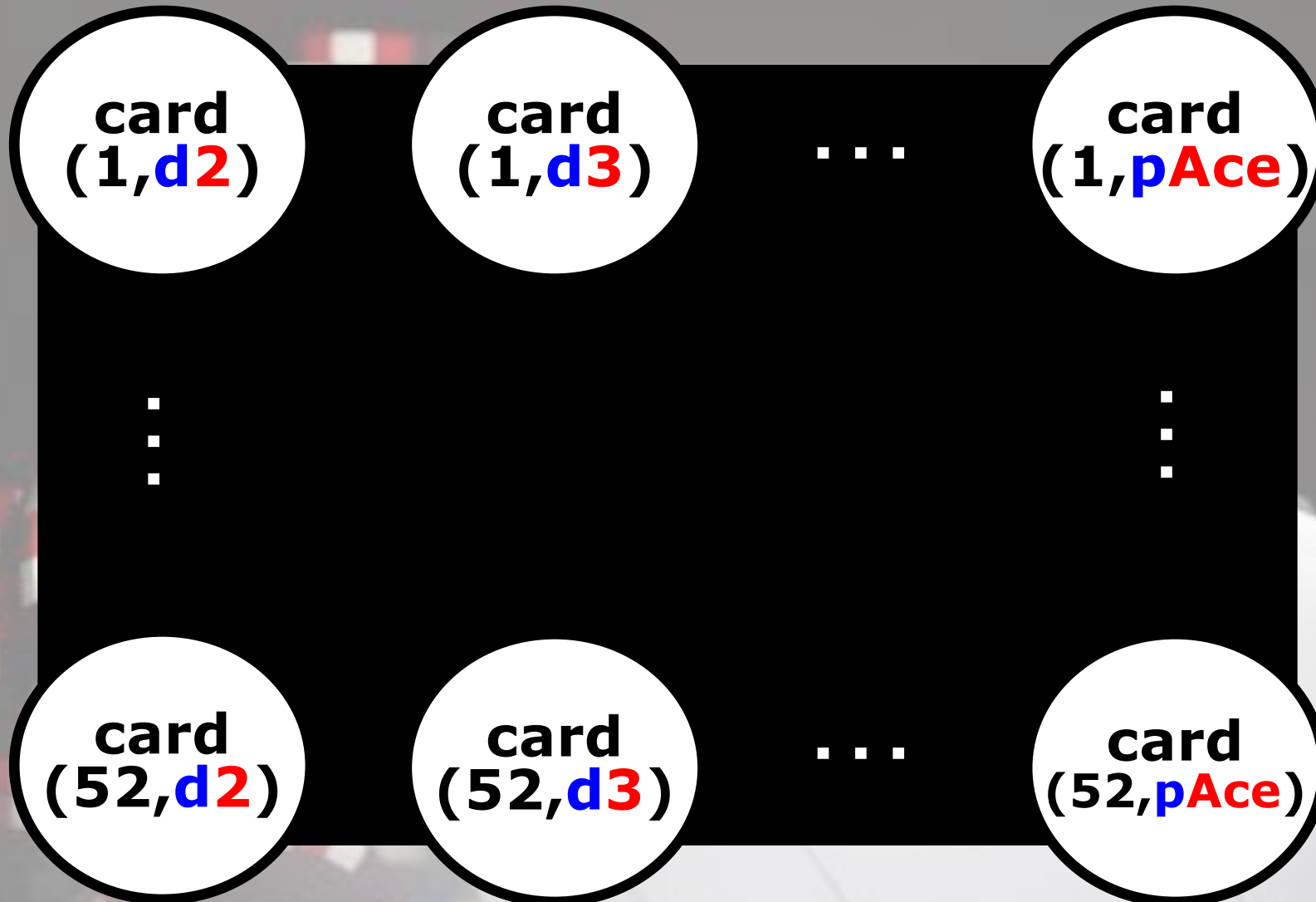
Guy van den Broeck
UCLA

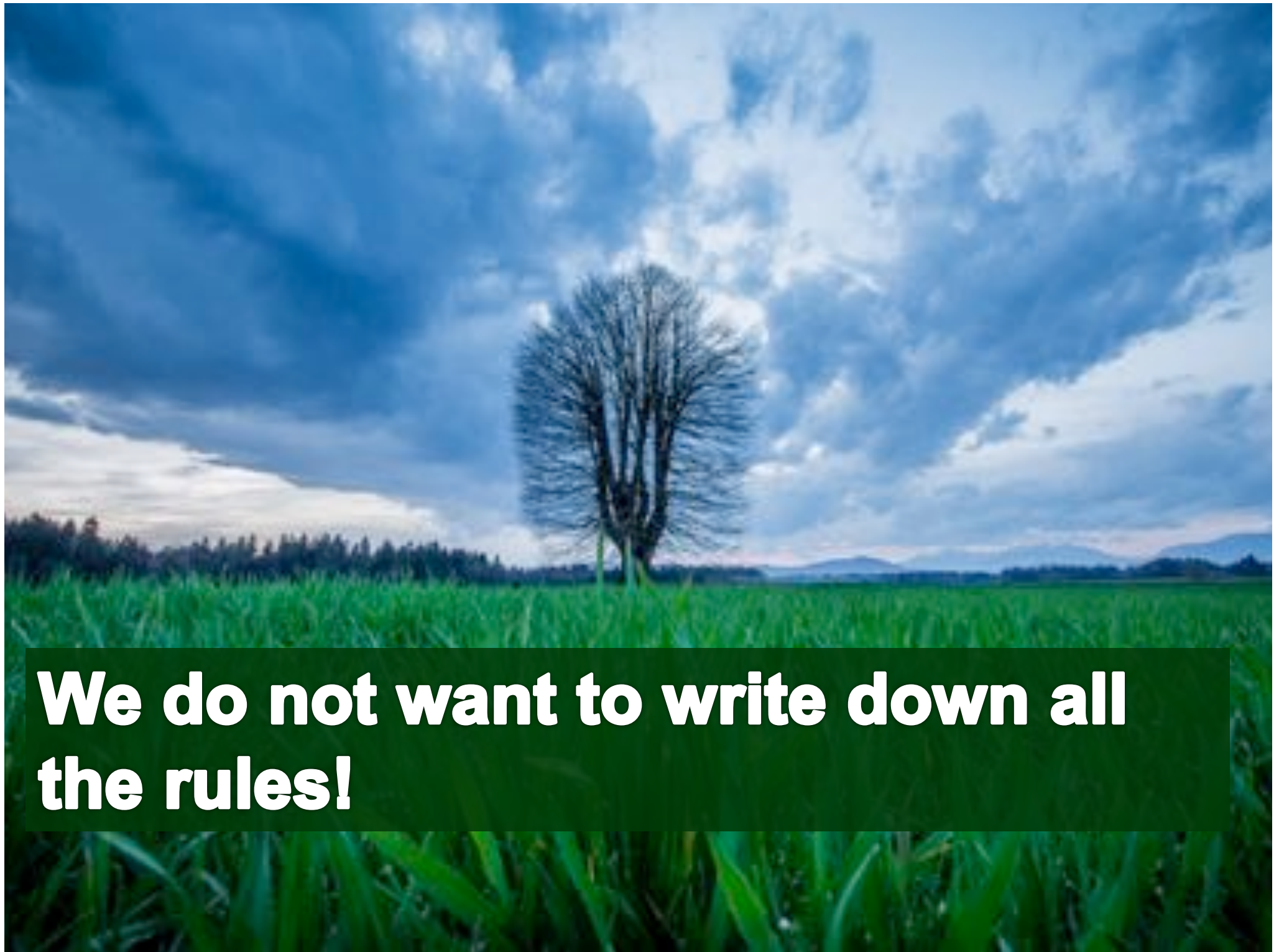


A simple example



Guy van den Broeck
UCLA





**We do not want to write down all
the rules!**

Faster modelling

Let's use programming abstractions such as e.g.

$w1: \forall p, x, y: \text{card}(P, X), \text{card}(P, Y) \Rightarrow x = y$

$w2: \forall c, x, y: \text{card}(X, C), \text{card}(Y, C) \Rightarrow x = y$

We do not want to write down all the rules!

A simple example



Guy van den Broeck
UCLA

card
(1, **d2**)

card
(1, **d3**)

...

card
(1, **pAce**)

∴ What about inference? ∴

card
(52, **d2**)

card
(52, **d3**)

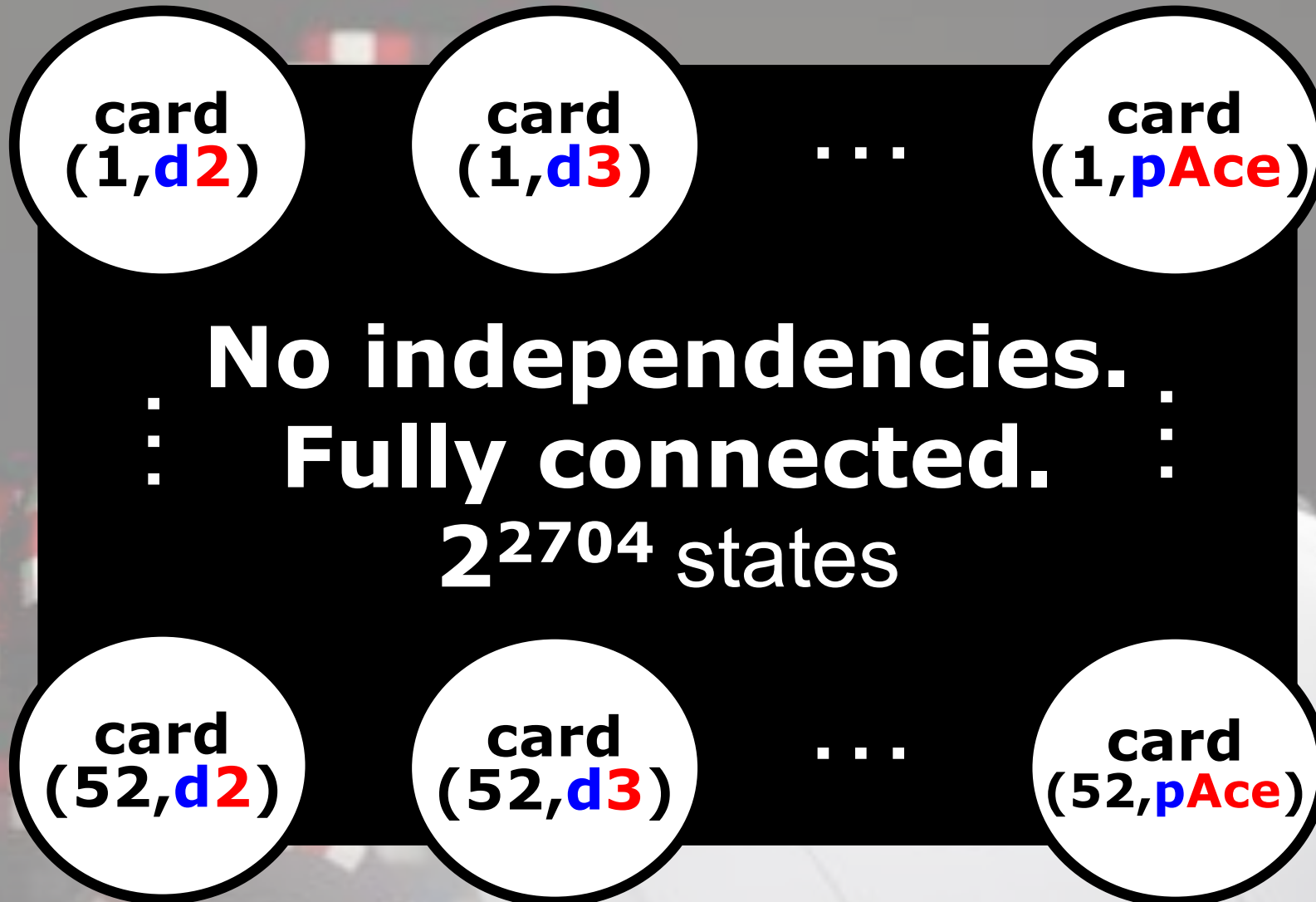
...

card
(52, **pAce**)

A simple example



Guy van den Broeck
UCLA



A simple example



Guy van den Broeck
UCLA

card
(1, **d2**)

card
(1, **d3**)

...

card
(1, **pAce**)

⋮ **A machine will not** ⋮
solve the problem ⋮

card
(52, **d2**)

card
(52, **d3**)

...

card
(52, **pAce**)

A landscape photograph featuring a single, tall, leafless tree standing in the center of a vibrant green field. The sky is filled with dramatic, layered clouds in shades of blue and grey, suggesting an overcast or stormy day. In the background, a dark line of trees and distant mountains are visible under the heavy sky.

What are we missing?

**Positions and cards are
exchangable but the machine is
not aware of these symmetries**

Faster modelling

Let's use programming abstractions together with symmetry- and language-aware solvers

Faster solvers

Positions and cards are exchangeable but the machine is not aware of these symmetries

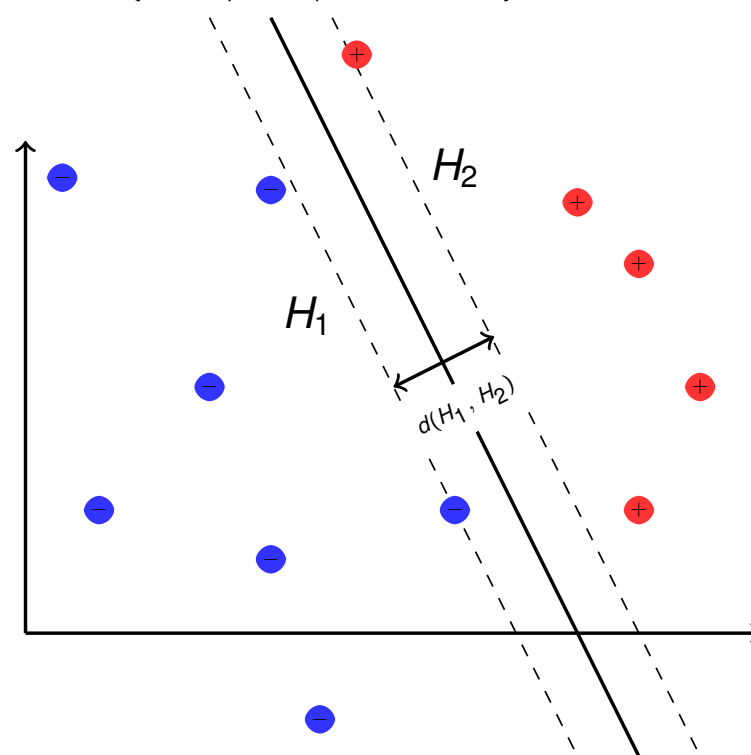
Let's make it more "optimization"-like
**Let's say we want to classify
publications into scientific disciplines**



Classification using LP SVMs

[Bennett '99; Mangasarian '99; Zhou, Zhang, Jiao '02, ...]

$$H^* = \{ \vec{x} \mid \langle \vec{x}, \vec{\beta} \rangle + \beta_0 = 0 \}$$



$$d(H_1, H_2) = \frac{2}{\|\vec{\beta}\|}$$

Replace l_2 - by l_1 -, l_∞ -norm in the standard SVM prog.



Relational Data and Program Abstractions

[Kersting, Mladenov, Tokmakov AIJ '15, Mladenov, Heinrich, Kleinhans, Gonsio, Kersting DeLBP '16]

Lifted LP-SVM

```
1 var pred/1;           #predicted label for unlabeled instances
2 var slack/1;         #the slacks
3 var coslack/2;      #s
4 var weight/1;       #the
5 var b/0;            #the
6 var r/0;            #marg
7
8 slack = sum{label(I)} slack(I);
9 coslack = sum{cite(I1,I2),label(I1),query(I2)} slack(I1,I2)
```

Logically parameterized LP variable
(set of ground LP variables)

Logically parameterized LP objective

```
12 #find the best margin. here the C's encode trade-off parameters
13 minimize: -r + C(1) * slack + C(2) * coslack;
```

Write down the LP-SVM in „paper form“.
The machine compiles it into solver form.




<http://www-ai.cs.uni-dortmund.de/weblab/static/RLP/html/>

RELOOP: A Toolkit for Relational Convex Optimization

Embedded within Python s.t. loops and rules can be used

```
23 #examples should be on the correct side of the hyperplane
24 subject to forall {I in label(I)}:
25     label(I)*(innerProd(I) + b) + slack(I) >= r;
26 #weights are between -1 and 1
27 subject to for
28 subject to : r
29 subject to forall {I in label(I)}: slack(I) >= 0, #slacks are positive
```

Logically parameterized LP constraint



But wait, publications are citing each other. OMG, I have to use graph kernels!

REALLY?

Relational Data and Program Abstractions

[Kersting, Mladenov, Tokmakov AIJ '15, Mladenov, Heinrich, Kleinhaus, Gonsio, Kersting DeLBP '16]

```
1 var pred/1;           #predicted label for unlabeled instances
2 var slack/1;          #the slacks
3 var coslack/2;        #slack between neighboring instances
4 var weight/1;         #the slope of the hyperplane
5 var b/0;              #the intercept of the hyperplane
6 var r/0;              #margin
7
8 slack = sum{label(I)} slack(I);
```

Lifted LP-SVM

Logical query defines scope of abstract collective constraint

```
slack(I1, I2)
slack(I1, I2)
```

code trade-off parameters

```
13 minimize: -r + C(I) * s + ... * coslack;
```

Citing papers share topics

Collective constraints

```
15 subject to forall {I in query} pred(I) = innerProd(I) + b;
16 #related instances should have the same labels.
17 subject to forall {I1, I2 in cite(I1, I2), label(I1), query(I2)}:
18   label(I1) * pred(I2) + slack(I1, I2) >= r;
19 #the symmetric case
20 subject to forall {I1, I2 in cite(I1, I2), label(I2), query(I1)}:
21   label(I2) * pred(I1) + slack(I1, I2) >= r;
```

No kernel, the structure is expressed within the constraints!

```
24 subject to forall {I in label(I)}:
25   label(I)*(innerProd(I) + b) + slack(I) >= r;
26 #weights are between -1 and 1
27 subject to forall {J in attribute(_, J)}: -1 <= weight(J) <= 1;
28 subject to : r >= 0;           #the margin is positive
29 subject to forall {I in label(I)}: slack(I) >= 0;       #slacks are positive
```

OK, we have now a high-level, declarative language for mathematical programming.

HOW CAN THE MACHINE  NOW HELP TO REDUCE THE SOLVER COSTS?



Lifted Mathematical Programming

Exploiting computational symmetries

[Mladenov, Ahmadi, Kersting AISTATS '12, Grohe, Kersting, Mladenov, Selman ESA '14,
Kersting, Mladenov, Tokmatov AIJ '17]



If exchanging two variables
preserves optimality, group
them together

automatically
compressed



Lifted Mathematical Programming

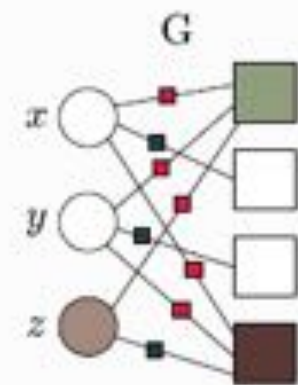
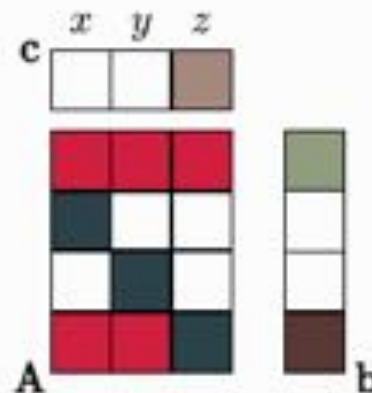
Exploiting computational symmetries

[Mladenov, Ahmadi, Kersting AISTATS '12, Grohe, Kersting, Mladenov, Selman ESA '14, Kersting, Mladenov, Tokmatov AIJ '17]

$$\max_{[x,y,z]^T \in \mathbb{R}^3} 0x + 0y + 1z$$

s.t.

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \leq \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$



View the mathematical program as a colored graph

Reduce the MP by running Weisfeiler-Lehman
on the MP-Graph

Weisfeiler-Lehman (WL) aka “naive vertex classification”

Basic subroutine for GI testing

Computes LP-relaxations of GA-ILP,

fractional automorphisms

Quasi-linear running time $O((n+m)\log(n))$ when
using asynchronous updates [Berkholz, Bonsma, Grohe ESA '13]

Part of graph tool SAUCY [See e.g. Darga, Sakallah, Markov DAC '08]

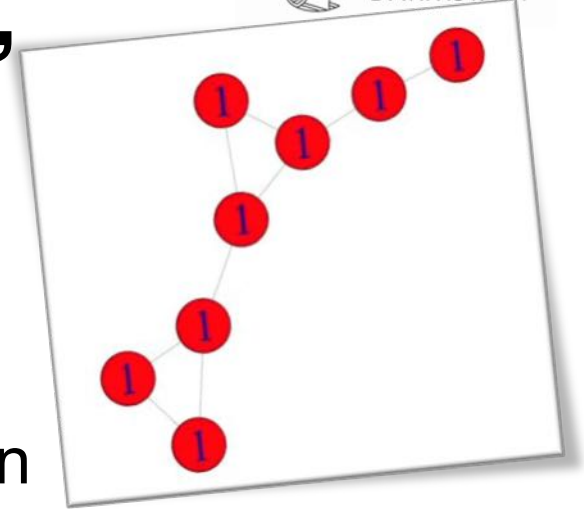
Has lead to highly performant graph kernels

[Shervashidze, Schweitzer, van Leeuwen, Mehlhorn, Borgwardt JMLR 12:2539-2561 '11]

Can be extended to weighted graphs/real-valued matrices

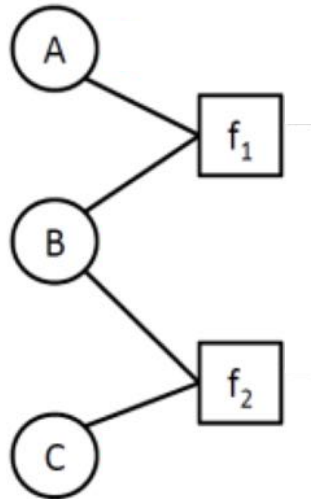
[Grohe, Kersting, Mladenov, Selman ESA '14]

Actually a Frank-Wolfe optimizer and can be viewed as
recursive spectral clustering [Kersting, Mladenov, Garnett, Grohe AAI '14]



Compression: Coloring the graph

[Kersting, Ahmadi, Natarajan UAI'09; Ahmadi, Kersting, Mladenov, Natarajan MLJ'13, Mladenov, Ahmadi, Kersting AISTATS '12, Grohe, Kersting, Mladenov, Selman ESA '14, Kersting, Mladenov, Tokmatov AIJ '17]



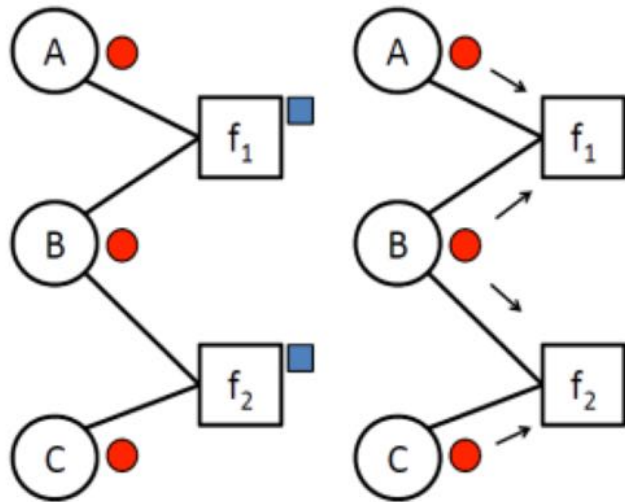
Color nodes initially with the same color, say **red**

Color factors distinctively according to their equivalences. For instance, assuming f_1 and f_2 to be identical and B appears at the second position within both, say **blue**



Compression: Pass colors around

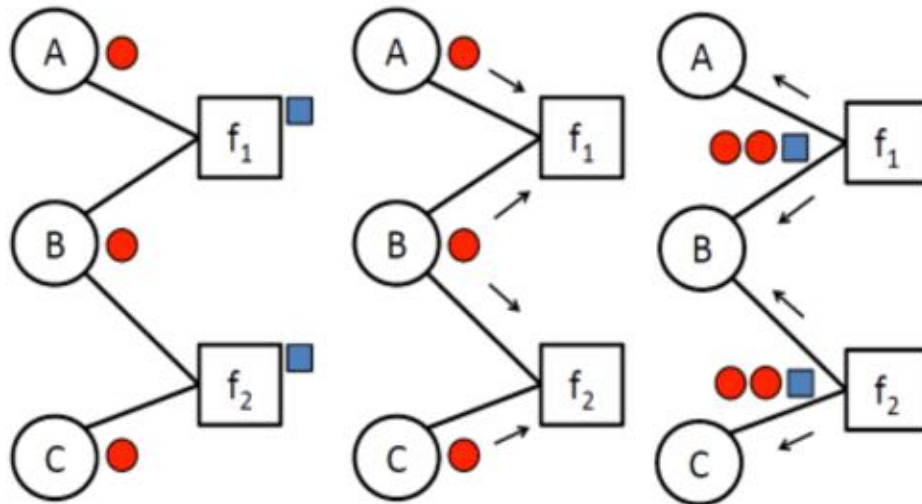
[Kersting, Ahmadi, Natarajan UAI'09; Ahmadi, Kersting, Mladenov, Natarajan MLJ'13, Mladenov, Ahmadi, Kersting AISTATS '12, Grohe, Kersting, Mladenov, Selman ESA '14, Kersting, Mladenov, Tokmatov AIJ '17]



1. Each factor collects the colors of its neighboring nodes

Compression: Pass colors around

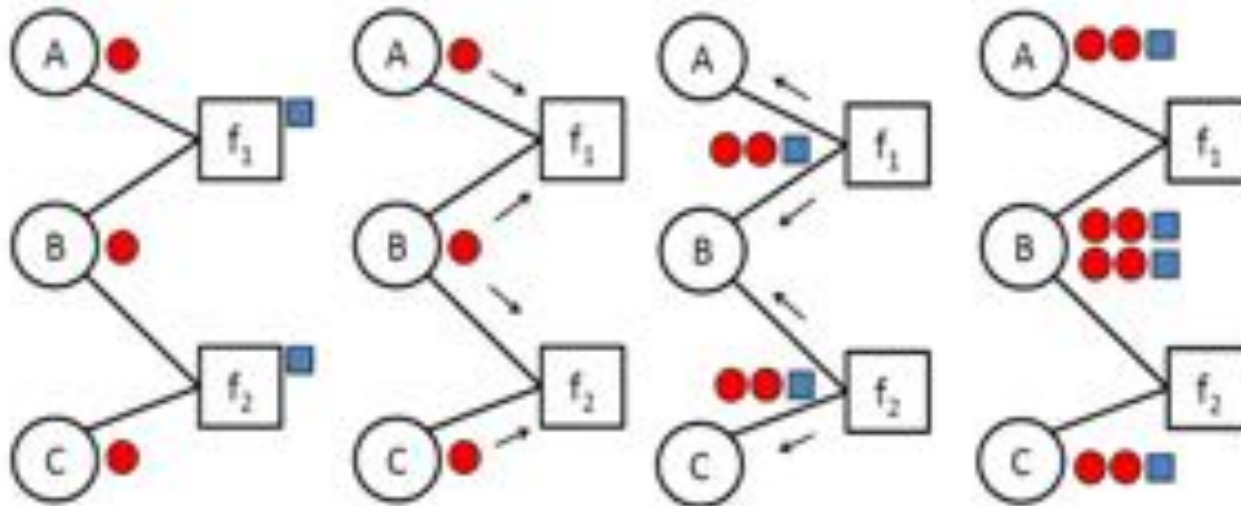
[Kersting, Ahmadi, Natarajan UAI'09; Ahmadi, Kersting, Mladenov, Natarajan MLJ'13, Mladenov, Ahmadi, Kersting AISTATS '12, Grohe, Kersting, Mladenov, Selman ESA '14, Kersting, Mladenov, Tokmatov AIJ '17]



1. Each factor collects the colors of its neighboring nodes
2. Each factor „signs“ its color signature with its own color

Compression: Pass colors around

[Kersting, Ahmadi, Natarajan UAI'09; Ahmadi, Kersting, Mladenov, Natarajan MLJ'13, Mladenov, Ahmadi, Kersting AISTATS '12, Grohe, Kersting, Mladenov, Selman ESA '14, Kersting, Mladenov, Tokmatov AIJ '17]

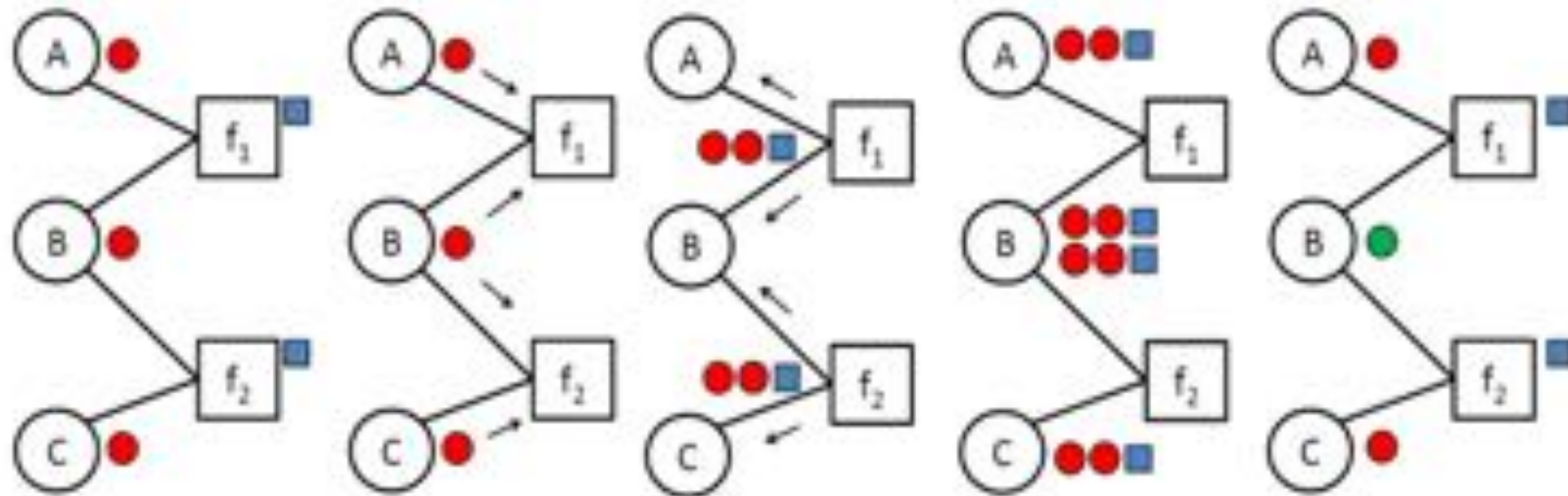


1. Each factor collects the colors of its neighboring nodes
2. Each factor „signs“ its color signature with its own color
3. Each node collects the signatures of its neighboring factors



Compression: Pass colors around

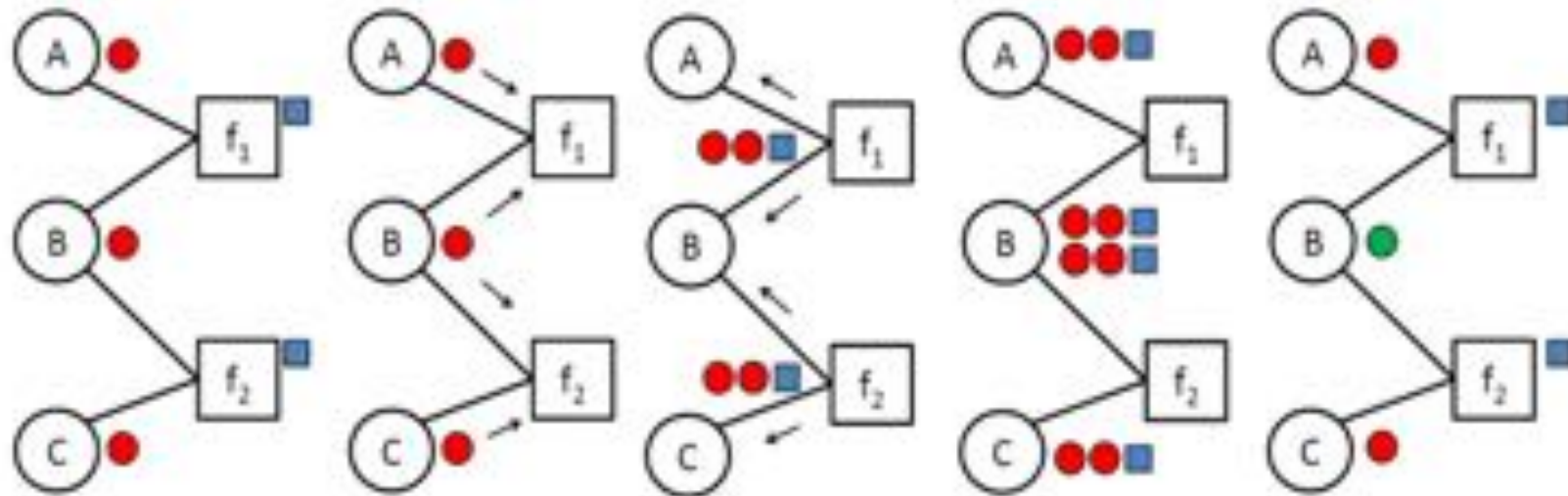
[Kersting, Ahmadi, Natarajan UAI'09; Ahmadi, Kersting, Mladenov, Natarajan MLJ'13, Mladenov, Ahmadi, Kersting AISTATS '12, Grohe, Kersting, Mladenov, Selman ESA '14, Kersting, Mladenov, Tokmatov AIJ '17]



1. Each factor collects the colors of its neighboring nodes
2. Each factor „signs“ its color signature with its own color
3. Each node collects the signatures of its neighboring factors
4. Nodes are recolored according to the collected signatures

Compression: Pass colors around

[Kersting, Ahmadi, Natarajan UAI'09; Ahmadi, Kersting, Mladenov, Natarajan MLJ'13, Mladenov, Ahmadi, Kersting AISTATS '12, Grohe, Kersting, Mladenov, Selman ESA '14, Kersting, Mladenov, Tokmatov AIJ '17]



1. Each factor collects the colors of its neighboring nodes
2. Each factor „signs“ its color signature with its own color
3. Each node collects the signatures of its neighboring factors
4. Nodes are recolored according to the collected signatures
5. If no new color is created stop, otherwise go back to 1

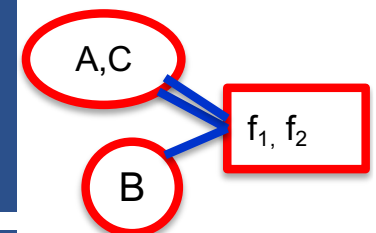
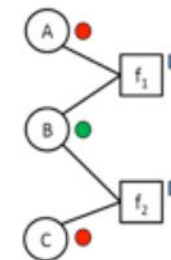
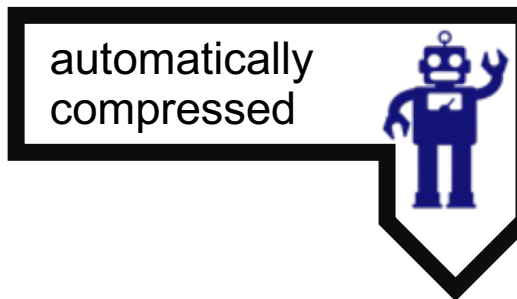
Lifted Mathematical Programming

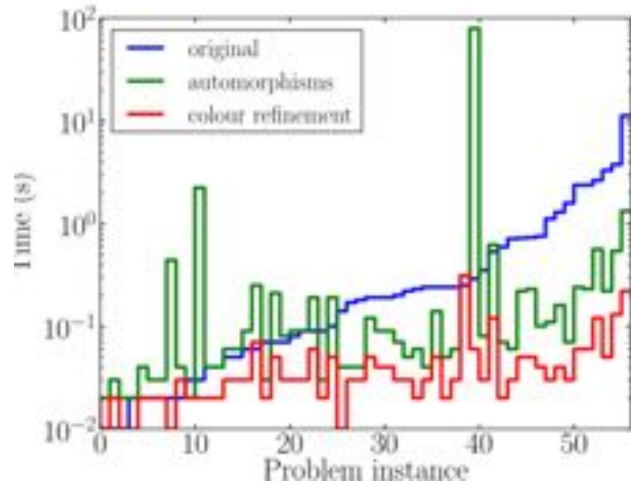
Exploiting computational symmetries

[Mladenov, Ahmadi, Kersting AISTATS '12, Grohe, Kersting, Mladenov, Selman ESA '14, Kersting, Mladenov, Tokmatov AIJ '17]

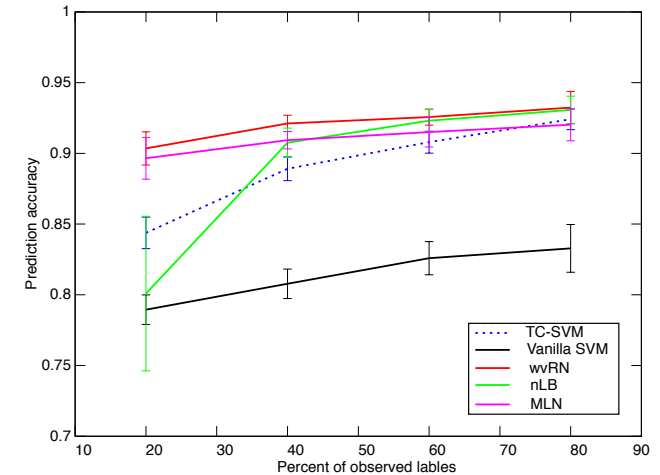


Weisfeiler-Lehman in quasi-linear time



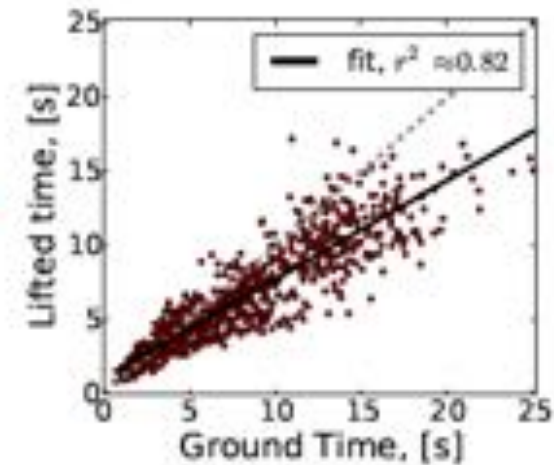
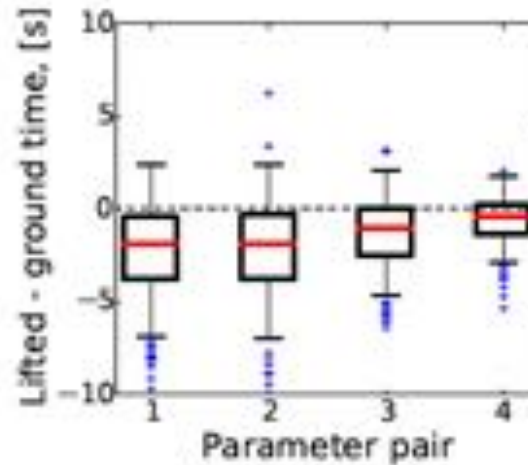
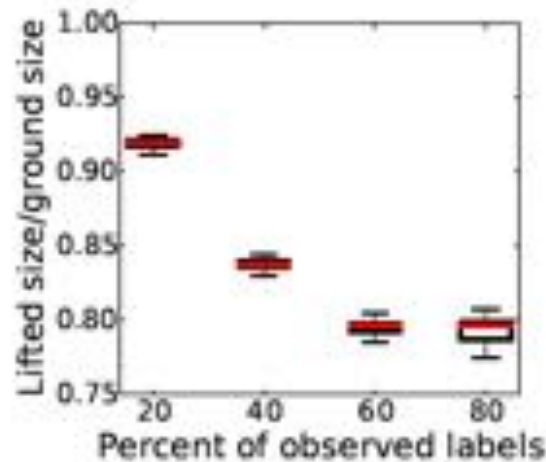


Margout's ILPs with symmetries (relaxed)



Collective Classification

Cora (most common vs. rest)



The more observed the more lifting

Faster end-to-end even in the light of Gurobi's fast pre-solving heuristics

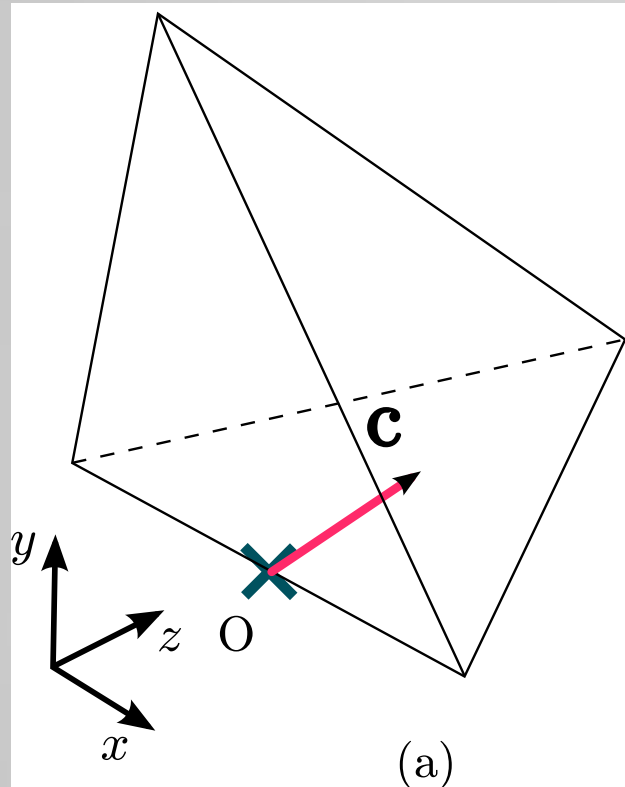




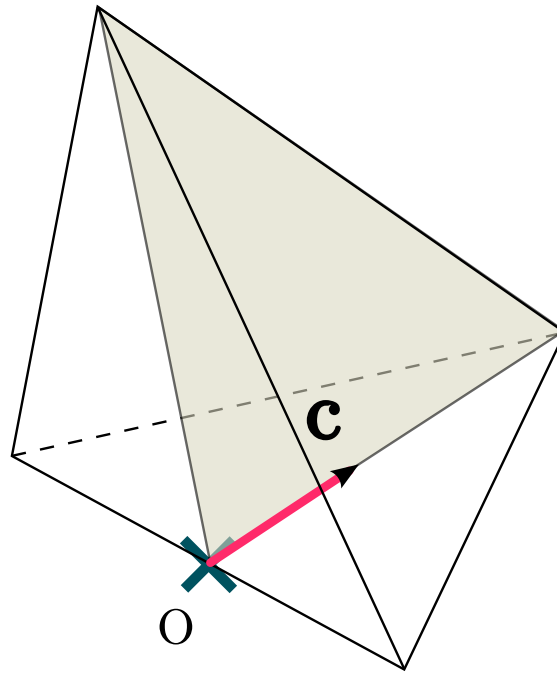
[Boyd, Diaconis, Parrilo, Xiao: Internet Mathematics 2(1):31-71'05]

As also noted by Stephen Boyd

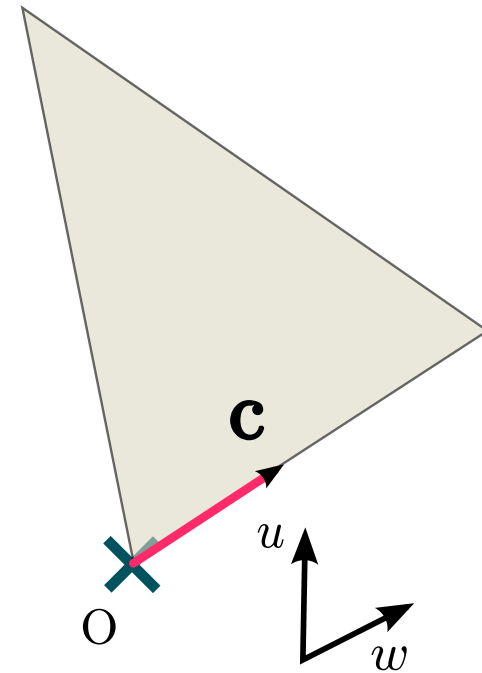
**Dense vs. sparse is not enough,
solvers need to be aware of
symmetries**



(a)



(b)



(c)

Feasible region
of LP and the
objective vectors

Span of the fractional
auto-morphism of the LP

Projections of the feasible
region onto the span of
the fractional auto-
morphism

Why does this work?

Holds also for Convex QPs

Mladenov, Kleinhans, Kersting AAAI '17

$$\begin{aligned} \mathbf{x}^* &= \arg \min_{\mathbf{x} \in \mathcal{D}} J(\mathbf{x}) \\ J(\mathbf{x}) &= \mathbf{x}^T Q \mathbf{x} + \mathbf{c}^T \mathbf{x} \\ \mathcal{D} &= \{\mathbf{x} : A \mathbf{x} \leq \mathbf{b}\} \end{aligned}$$

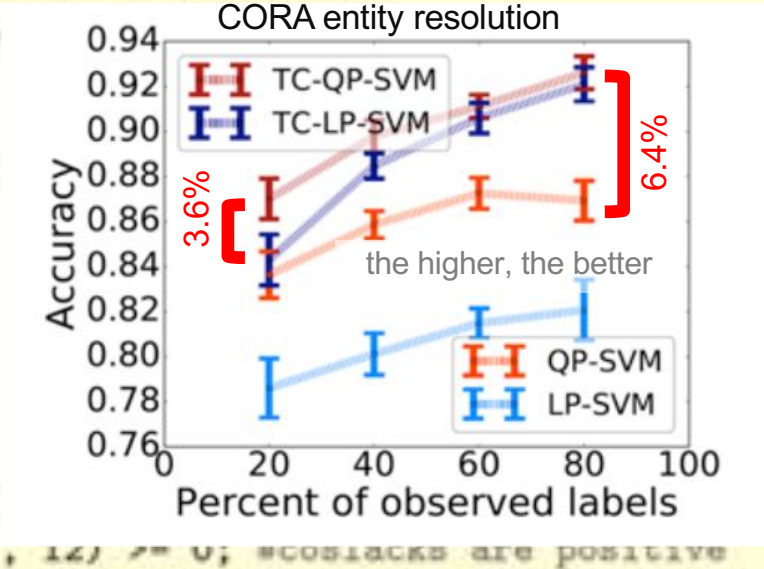
```
#QUADRATIC OBJECTIVE
minimize: sum{J in feature(I,J)} weight(J)**2 + c1 * sla

#labeled examples should be on the correct side
subject to forall {I1, I2 in linked(I1, I2)}: labeled(I1, I2)

#slacks
subject to forall {I1, I2 in linked(I1, I2)}: slack(I1, I2)

#TRANSDUCTIVE PART
#cited instances should have the same labels.
subject to forall {I1, I2 in linked(I1, I2)}: labeled(I1, I2)
subject to forall {I1, I2 in linked(I1, I2)}: coslack(I1, I2) == 0; #coslacks are positive
```

On par with state-of-the-art by just four lines of code



Papers that cite each other should be on the same side of the hyperplane

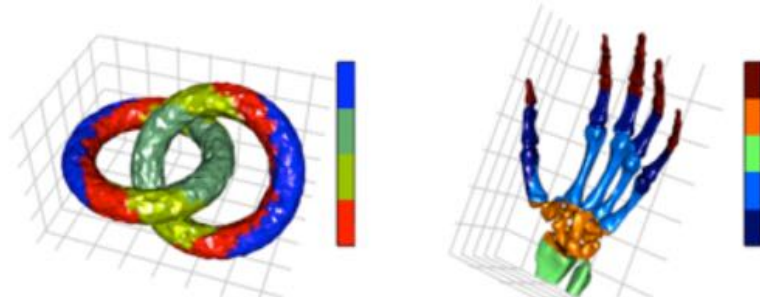
Reduce the QP by running Weisfeiler-Lehman on the QP-Graph

$$\max_{[x,y,z]^T \in \mathbb{R}^3} 0x + 0y + 1z$$

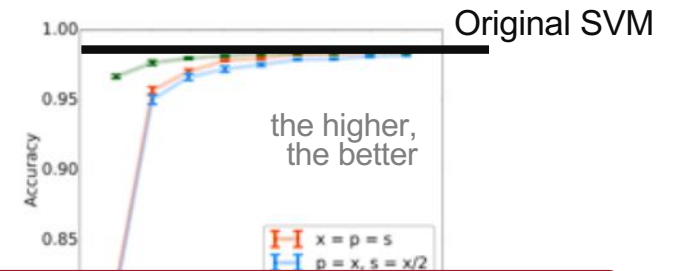
$$\text{s.t.} \quad -1z^2 - 2x^2 - 2y^2 + 1xy + 1yx$$

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \leq \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

Approximately Lifted SVM:
Cluster data points via K-means using sorted distance vectors. Solve SVM on cluster representatives only

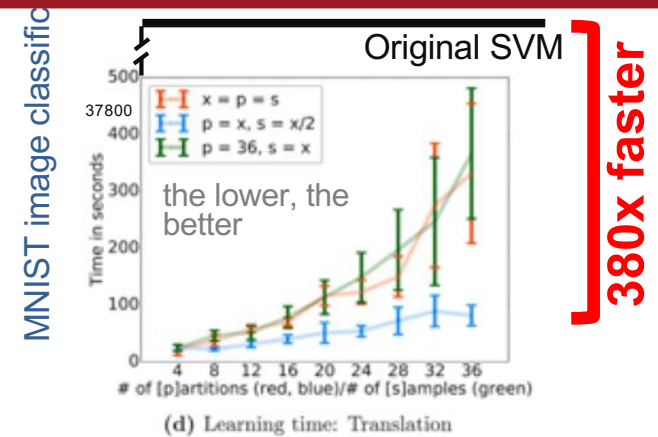


Symmetry-based Data Programming:
fractional autom. of label-preserving data transformations



Same should work for deep networks

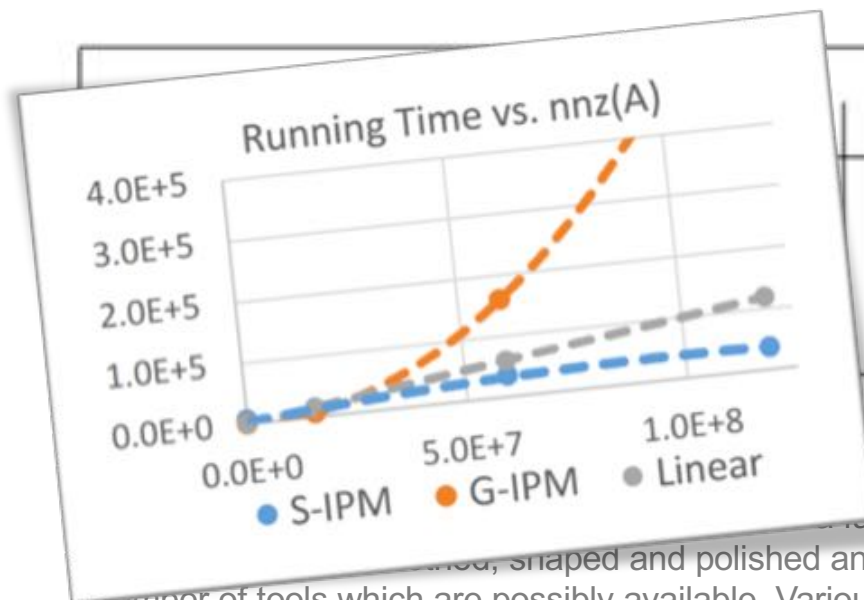
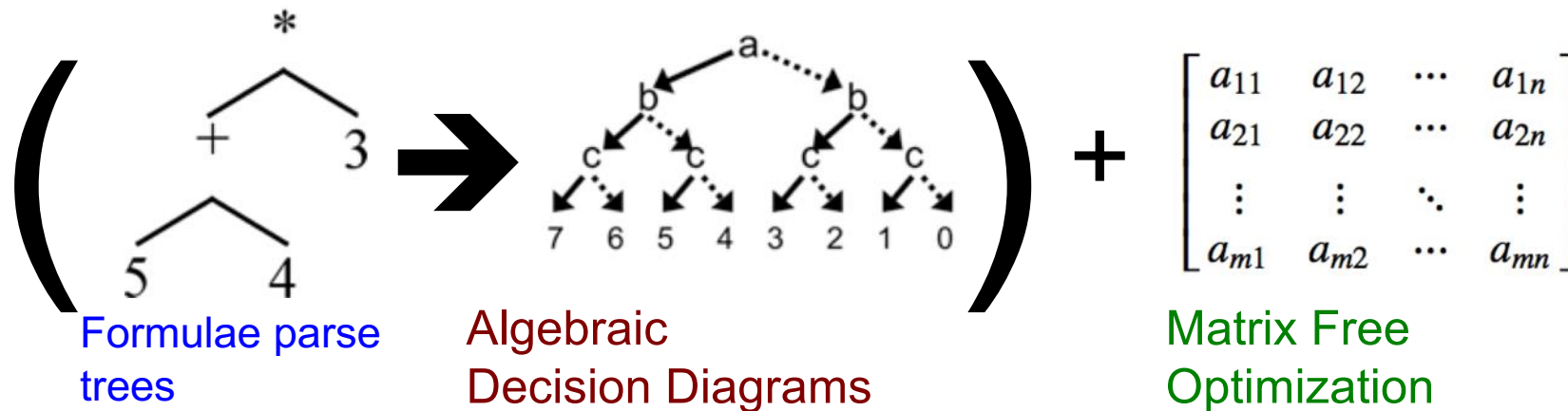
PAC-style generalization bound:
the approximately lifted SVM will very likely have a small expected error rate if it has a small empirical loss over the original dataset.



Similar predictive performance but 47x faster



And, there are other “-02”, “-03”, ... flags, e.g symbolic-numerical interior point solvers



All this opens the general machine learning toolbox for declarative machines: feature selection, least-squares regression, label propagation, ranking, collaborative filtering, community detection, deep learning, ...

... shaped and polished and possibly drilled before painting, each of which actions require a number of tools which are possibly available. Various painting and connection methods are represented, each having an effect on the quality of the job, and each requiring tools. Rewards (required quality) range from 0 to 10 and a discounting factor of 0.9 was used used

There are strong invests into probabilistic programming

RelationalAI, Apple, Microsoft and Uber are investing hundreds of millions of US dollars



Since we need languages for Systems AI, the computational and mathematical modeling of complex AI systems.

[Laue et al. NeurIPS 2018; Kordjamshidi, Roth, Kersting:
“Systems AI: A Declarative Learning Based Programming
Perspective.” IJCAI-ECAI 2018]



Eric Schmidt, Executive Chairman, Alphabet Inc.: Just Say "Yes", Stanford Graduate School of Business, May 2, 2017. <https://www.youtube.com/watch?v=vbb-AjiXyh0>.

Overall, AI/ML/DS indeed refine “formal” science, but ...

- **AI is more than deep neural networks.** Probabilistic and causal models are whiteboxes that provide insights into applications
- **AI is more than a single table.** Loops, graphs, different data types, relational DBs, ... are central to data science and high-level programming languages for DS help to capture this complexity
- **AI is more than just Machine Learners and Statisticians**

Learning-based programming offers a framework for building systems that help to go beyond, democratize, and even automate traditional AI/ML/DS

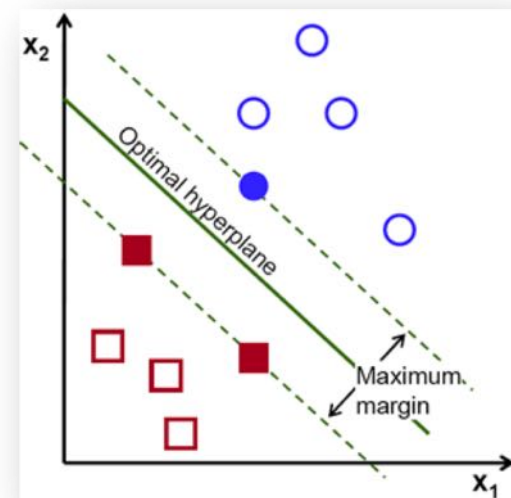
Not every Data Science machine is generative

$$\min_{\mathbf{w}, b, \xi} \mathcal{P}(\mathbf{w}, b, \xi) = \frac{1}{2} \mathbf{w}^2 + C \sum_{i=1}^n \xi_i$$

subject to $\begin{cases} \forall i & y_i(\mathbf{w}^\top \Phi(\mathbf{x}_i) + b) \geq 1 - \xi_i \\ \forall i & \xi_i \geq 0 \end{cases}$

Not everyone likes to turn math into code

Support Vector Machines
Cortes, Vapnik MLJ 20(3):273-297, 1995



High-level Languages for Mathematical Programs

Write down SVM in „paper form.“ The machine compiles it into solver form.

```
#QUADRATIC OBJECTIVE
minimize: sum{J in feature(I,J)} weight(J)**2 + c1 * slack + c2 * coslack;

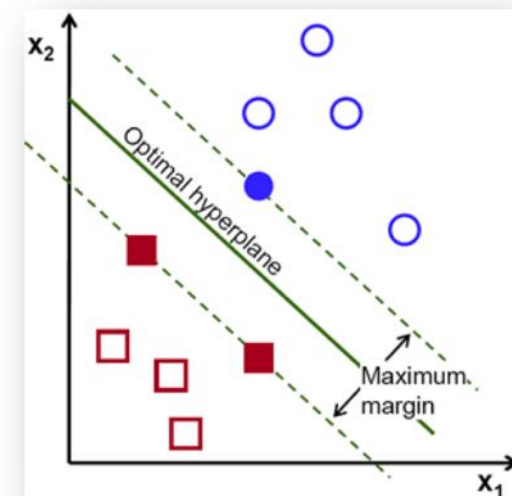
#labeled examples should be on the correct side
subject to forall {I in labeled(I)}: labeled(I)*predict(I) >= 1 - slack(I);

#slacks are positive
subject to forall {I in labeled(I)}: slack(I) >= 0;
```

Embedded within
Python s.t. loops and
rules can be used

reloop

RELOOP: A Toolkit for Relational Convex Optimization



Support Vector Machines

Cortes, Vapnik MLJ 20(3):273-297, 1995



In general, computing the exact posterior is intractable, i.e., inverting the generative process to determine the state of latent variables corresponding to an input is time-consuming and error-prone.

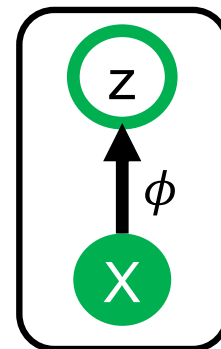
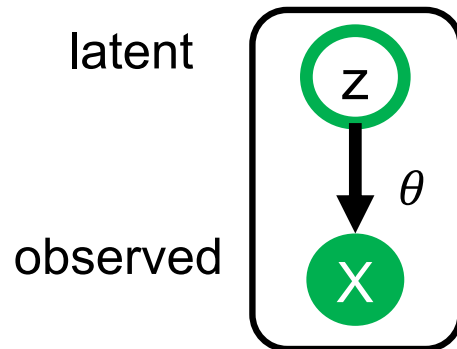
Deep Probabilistic Programming

```
import pyro.distributions as dist

def model(data):
    # define the hyperparameters that control the beta prior
    alpha_theta = torch.tensor(10.0)
    beta_theta = torch.tensor(10.0)
    # sample f from the beta prior
    f = pyro.sample("latent_fairness", dist.Beta(alpha_theta, beta_theta))
    # loop over the observed data
    for i in range(len(data)):
        # observe datapoint i using the bernoulli
        # likelihood Bernoulli(f)
        pyro.sample("obs_{}".format(i), dist.Bernoulli(f), obs=data[i])
```

```
def guide(data):
    # register the two variational parameters with Pyro.
    alpha_q = pyro.param("alpha_q", torch.tensor(15.0),
                        constraint=constraints.positive)
    beta_q = pyro.param("beta_q", torch.tensor(15.0),
                       constraint=constraints.positive)
    # sample latent_fairness from the distribution Beta(alpha_q, beta_q)
    pyro.sample("latent_fairness", dist.Beta(alpha_q, beta_q))
```

(2) Ease the implementation by some high-level, probabilistic programming language



Deep Neural Network



(1) Instead of optimizing variational parameters for every new data point, use a deep network to predict the posterior given X [Kingma, Welling 2013, Rezende et al. 2014]



UBER AI Labs



UNIVERSITY OF CAMBRIDGE



Max Planck Institute for Intelligent Systems



TECHNISCHE UNIVERSITÄT DARMSTADT

[Stelzner, Molina, Peharz, Vergari, Trapp, Valera, Ghahramani, Kersting ProgProb 2018]

Sum-Product Probabilistic Programming

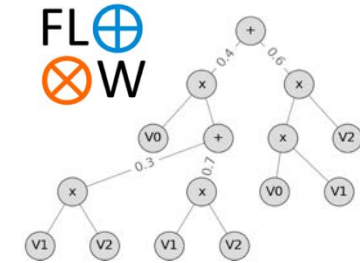
```
import pyro.distributions as dist

def model(data):
    # define the hyperparameters that control the beta prior
    alpha0 = torch.tensor(10.0)
    beta0 = torch.tensor(10.0)
    # sample f from the beta prior
    f = pyro.sample("latent_fairness", dist.Beta(alpha0, beta0))
    # loop over the observed data
    for i in range(len(data)):
        # observe datapoint i using the bernoulli
        # likelihood Bernoulli(f)
        pyro.sample("obs_{}".format(i), dist.Bernoulli(f), obs=data[i])
```

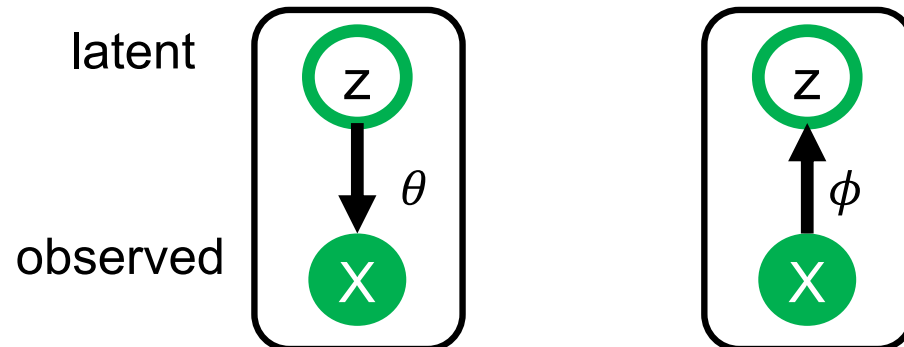
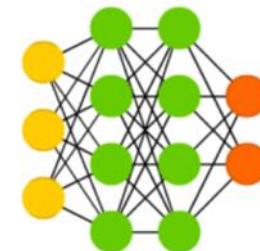
```
def guide(data):
    # register the two variational parameters with Pyro.
    alpha_q = pyro.param("alpha_q", torch.tensor(15.0),
                        constraint=constraints.positive)
    beta_q = pyro.param("beta_q", torch.tensor(15.0),
                       constraint=constraints.positive)
    # sample latent_fairness from the distribution Beta(alpha_q, beta_q)
    pyro.sample("latent_fairness", dist.Beta(alpha_q, beta_q))
```

(2) Ease the implementation by some high-level, probabilistic programming language

Sum-Product Network



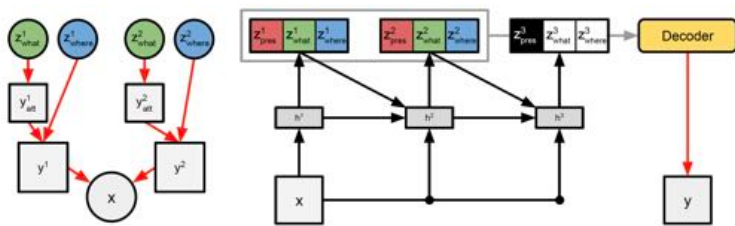
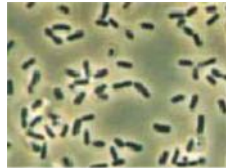
Deep Neural Network



(1) Instead of optimizing variational parameters for every new data point, use a deep network to predict the posterior given X [Kingma, Welling 2013, Rezende et al. 2014]

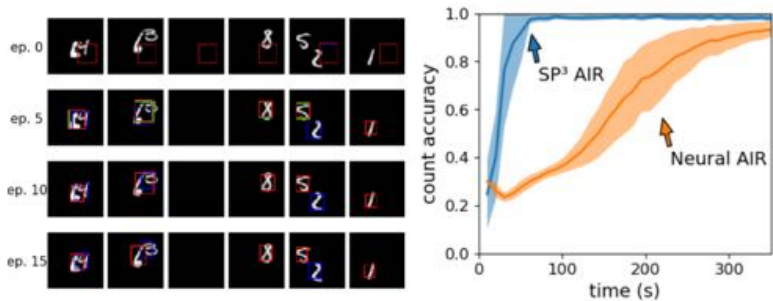
Unsupervised scene understanding

Consider e.g. unsupervised scene understanding using a generative model



[Attend-Infer-Repeat (AIR) model, Hinton et al. NIPS 2016]

Sum-Product Probabilistic Programming:
Making machine learning and data science easier [Stelzner, Molina, Peharz, Vergari, Trapp, Valera, Ghahramani, Kersting ProgProb 2018]



Probabilistic Programming:
Easier modelling by programming generative models in a high-level, prob. language

```
def prior_step(t):
    # Sample object pose. This is a 3-dimensional vector representing x,y position and size.
    z_where = pyro.sample('z_where_{}'.format(t),
        dist.normal,
        z_where_prior_mu, z_where_prior_sigma)

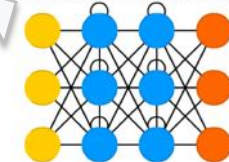
    # Sample object code. This is a 50-dimensional vector.
    z_what = pyro.sample('z_what_{}'.format(t),
        dist.normal,
        z_what_prior_mu, z_what_prior_sigma)

    y_att = decode(z_what) # Map latent code to pixel space using the neural net
```

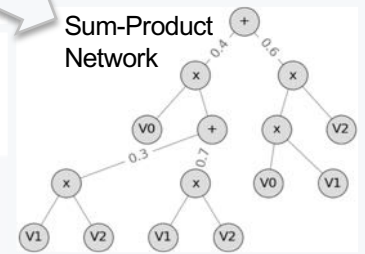


Deep Probabilistic Prog.:
Modelling and inference might be hard, so use a deep neural network for it

Recurrent Neural Network (RNN)



Sum-Product Network



Use deep probabilistic models that feature tractable, deterministic inference

```
from spn.structure.leaves.parametric.Parametric import Categorical
from spn.structure.Base import Sum, Product
from spn.structure.base import assign_ids, rebuild_scopes_bottom_up

p0 = Product(children=[Categorical(p=[0.3, 0.7], scope=1), Categorical(p=[0.4, 0.6], scope=2)])
p1 = Product(children=[Categorical(p=[0.5, 0.5], scope=1), Categorical(p=[0.6, 0.4], scope=2)])
s1 = Sum(weights=[0.3, 0.7], children=[p0, p1])
p2 = Product(children=[Categorical(p=[0.2, 0.8], scope=0), s1])
p3 = Product(children=[Categorical(p=[0.2, 0.8], scope=0), Categorical(p=[0.3, 0.7], scope=1)])
p4 = Product(children=[p3, Categorical(p=[0.4, 0.6], scope=2)])
spn = Sum(weights=[0.4, 0.6], children=[p2, p4])

assign_ids(spn)
rebuild_scopes_bottom_up(spn)

return spn
```



Actually, the main idea is to replace the VAEs within AIR by SPNs

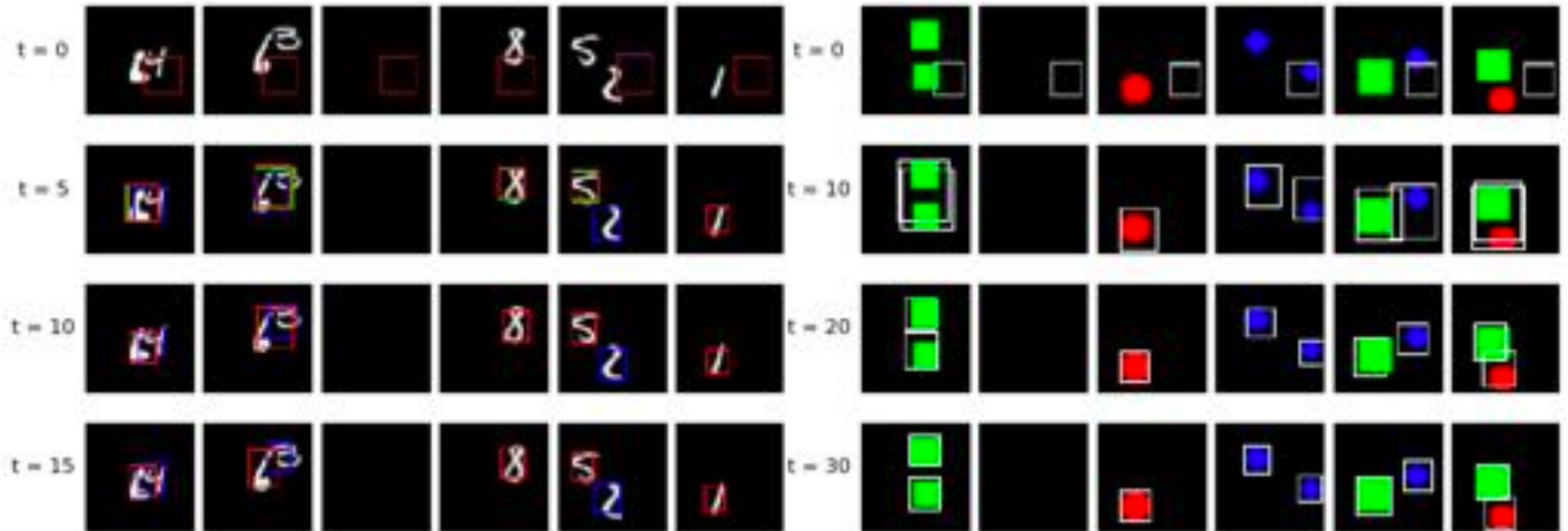


- infinite mixture model
- intractable density
- intractable posterior



- “large” but finite mixture model
- tractable density
- tractable marginals [Peharz et al., 2015]
- tractable posterior [Vergari et al., 2017]

Sum-Product Attent-Infer Repeat



[Stelzner, Peharz, Kersting 2019]



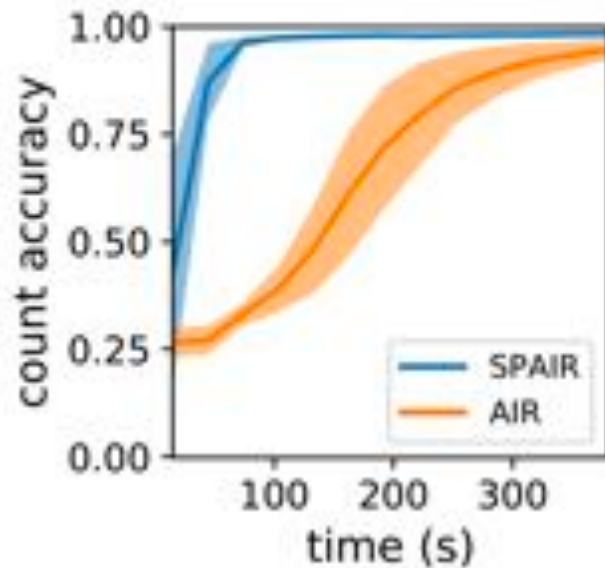
UNIVERSITY OF
CAMBRIDGE



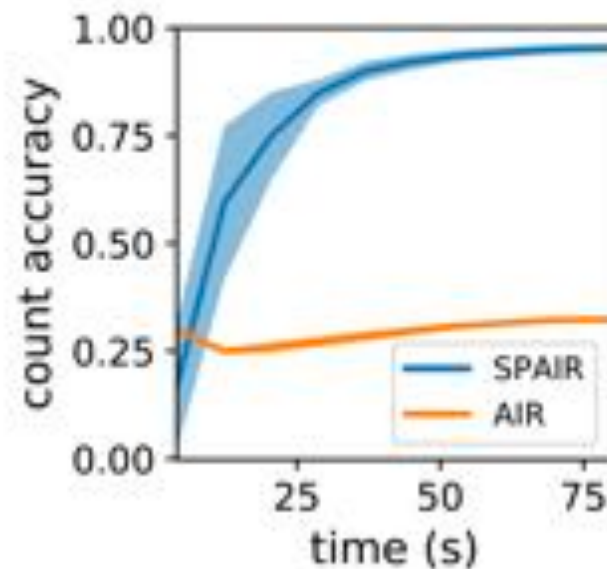
TECHNISCHE
UNIVERSITÄT
DARMSTADT

Sum-Product Attent-Infer Repeat

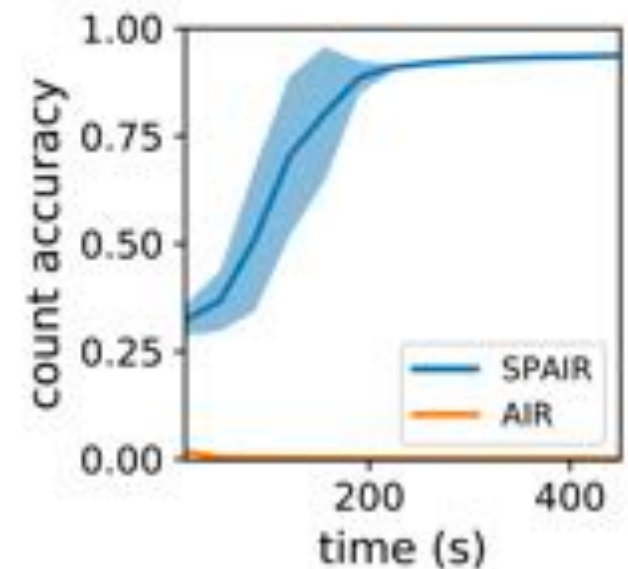
Multi-MNIST



Sprites



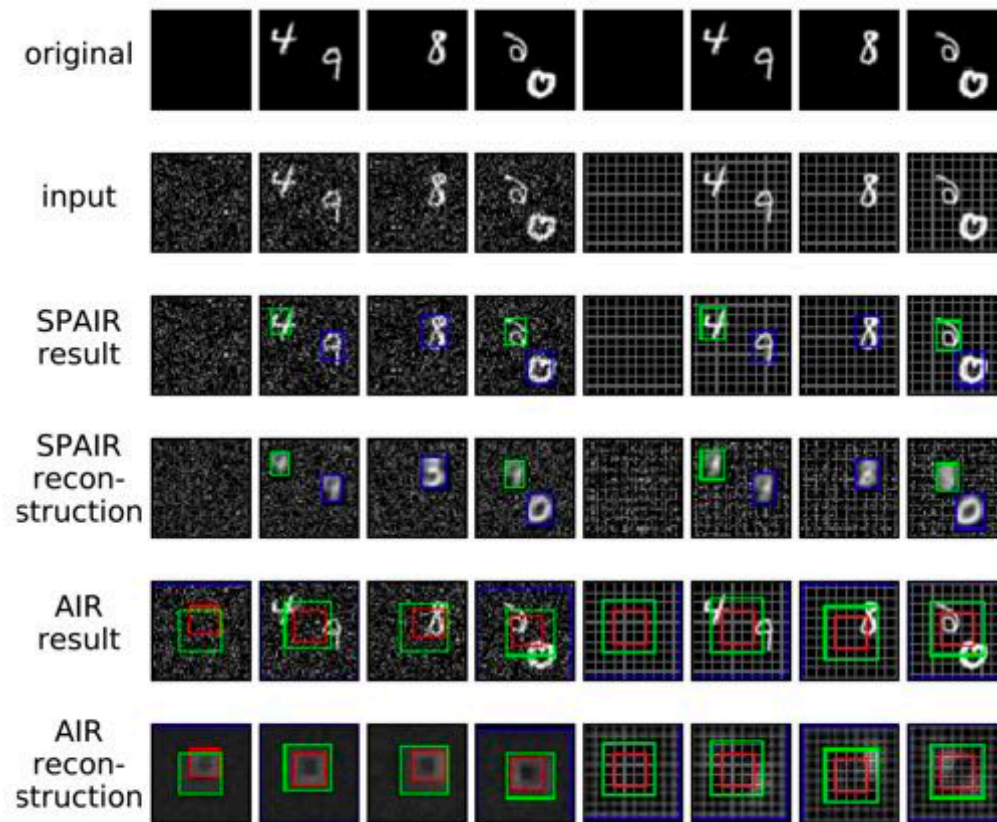
Noisy MNIST



[Stelzner, Peharz, Kersting 2019]



Sum-Product Attent-Infer Repeat



[Stelzner, Peharz, Kersting 2019]